Noname manuscript No.
(will be inserted by the editor)

Simplifying the Role of Signals in Tile Self-assembly

Lila Kari · **Amirhossein Simjour**

Abstract Sending signals through DNA-based structures is one of the methods used to enhance the capabilities of DNA self-assembly systems. Signal Tile Assembly Models at temperature one, in supertile-to-supertile attachment mode, have been showed to have universal computational power. We introduce a simplified signal tile assembly model, in one-tile-at-a-time attachment mode, and where signals can only be used to deactivate glues. We prove that such a simplified system at temperature one can still simulate a Turing machine. We also present a simplified signal tile assembly system, in supertile-to-supertile attachment mode, that assembles a thin, $N \times N!$, rectangle and has tile complexity $O(\log N)$. This result is an improvement over the tile complexity of existing models for thin rectangle self-assembly.

1 Introduction

A self-assembly system is an autonomous system whereby small components attach to each other via local interactions, in order to build a larger structure. Many examples of self-assembly systems exist in nature, an ubiquitous example being molecules that bind to each other via chemical bonds to form macromolecules or

L. Kari
Department of Computer Science
University of Western Ontario
E-mail: lila@csd.uwo.ca

School of Computer Science University of Waterloo
Waterloo, ON
N2L 3G1
Canada

This research was supported by Natural Science and Engineering Council of Canada (NSERC) Discovery Grant R2824A01 to L.K.

A. Simjour
Department of Computer Science
University of Western Ontario
E-mail: asimjour@csd.uwo.ca

crystals. DNA-based self-assembly was introduced by Seeman [15] in 1982, when he designed a nanoscale DNA complex that could attach to four similar DNA complexes via DNA Watson-Crick complementarity. The self-assembly of many such DNA complexes resulted into a two-dimensional DNA lattice. Afterwards [17] [18] introduced rectangular DNA complexes called *tiles*, with a "sticky end" (single-stranded DNA sequences) at each corner, called "glue". In this framework, a tile could attach to another tile if the sticky ends representing their corresponding corners were complementary DNA sequences. Winfree [16] introduced the abstract Tile Assembly Model (aTAM) as a theoretical model to describe DNA self-assembly systems. In aTAM, labelled unit squares with coloured edges are used to represent DNA tiles. The colours (labels) on the edges represent the glues of the DNA tiles. In addition to the labels on its edges, each tile itself can have a label. In aTAM, each glue has a "strength" (a positive integer value), and a single tile can attach to an already assembled structure if the sum of the strengths of the glues on the edges abutting the structure exceeds an *a priori* given positive numerical constant, called "temperature".

Following Winfree's aTAM, several other models were proposed for DNA self-assembly systems. For example, Aggarwal et al. [1] introduced a model that allows changes of the temperature during the assembly process, as well as a model that allows attachments of structures composed of multiple tiles. Aggarwal et al. [1] also compared different self-assembly models based on the number of different tile types (called *tile complexity* [13]) needed to construct an arbitrary rectangle. Reif et al. [12] investigated the possibility of tile detachment, and introduced a graph model as a general model for non-square tiles. Padilla et al. [9] introduced tiles that not only have multiple glues on each edge, but also have a limited ability to communicate with the tiles that are attached to them, via signals. The authors showed how signals can be implemented experimentally so as to be used to activate or deactivate glues. Doty et al. [6] expanded the aTAM model by adding the notion of repellent (negative-valued) glues. The authors showed that such a model can simulate a Turing machine using a smaller number of tiles compared to the original aTAM model. Other self-assembly models exist, see [11] for a review.

Padilla et al. [10] introduced the Signal Tile Assembly Model (STAM) as a mathematical model for tiles with the ability of sending signals that can activate and deactivate glues. STAM uses as underlying model the 2-Handed Assembly Model (2HAM) [5], where two structures composed of multiple tiles can attach to each other if the strength of the glues at the abutting edges of the structures exceed the temperature (as opposed to aTAM, where a single tile attaches to the growing structure, at every time step).

Padilla et al. [10] proved that, for some specific shapes, the use of STAM can reduce the tile complexity of the computation. Recently, Fochtman et al. [7] showed that STAM can be simulated by three-dimensional 2HAM. Keenan et al. [8] investigated the usability of STAM in the replication of some patterns (rectangular structures with coloured tiles which form a pattern). The construction has some limitations, e.g., patterns are hole-free, and all tiles have to be able to support a set of predefined signals to make replication possible.

In this paper, we introduce DTAM (Detachable Tile Assembly Model), which is a simplified version of STAM that is based on the 2-HAM model, and uses only glue-deactivating signals (instead of signals that can both activate and deactivate glues). We also introduce SDTAM (Simplified Detachable Tile Assembly Model), which is a further simplified version of DTAM wherein the attachment of only a single tile at each step is allowed. One of our main results shows a simulation of an arbitrary Turing machine by an SDTAM at temperature one (Theorem 01), showing thus that SDTAM can achieve universal computational power in spite of being a simplified version of STAM. Moreover, the Turing-simulating SDTAM we contruct utilizes at most one signal per tile, and signals travel through only one tile before deactivating a glue, both of which could have implications for the practical implementations of such signal-based self-assembly systems. Our second result presents a DTAM construction of a "thin rectangle", of size $N \times N!$, that uses only $O(\log N)$ tiles. This is an improvement over the tile complexity of existing models, the best of which use $O(\log N!/\log\log N!)$ tiles to build the same rectangle [1].

The paper is organized as follows. Section 2 introduces the formal definitions of DTAM and SDTAM. In Section 3 we prove that SDTAM can simulate a Turing machine - the construction is based on simulating a deterministic zig-zag tile assembly system (known to be Turing universal [4]) by an SDTAM at temperature 1. Section 4 presents the construction of an $N \times N!$ rectangle using DTAM, and calculates its improved tile complexity, and Section 5 presents the conclusions.

2 The Detachable Tile Assembly Model

Informal Description of STAM

The Signal Tile Assembly Model (STAM)[10] is a tile assembly model based on 2HAM, wherein each tile possesses a set of glues on each edge (instead of one glue per edge, like in aTAM and 2HAM), and glues can be activated or deactivated by *signals*. In STAM, each glue on an edge can be in one of three states: *latent*, *on*, or *off*. Only a glue that is active (it is in the *on* state) can contribute to attaching the tile to another tile with an identical active glue on the abutting edge. If the state of a glue is *off* or *latent*, the glue is inactive and it does not have any attachment power. In order to change the states of the glues, signals are used.

Intuitively, a signal is a mapping associated to a given tile that assigns to a glue on an edge a set of changes in the state of the glues on the other edges. For example, assume that tile t has glue g_e on the East edge, glue g_s on the South edge, and glue g_n on the North edge. Also assume that all these glues are *on*, and assume that there is a signal on the East side of the tile t that assigns a change of the state of the glue g_s to *off*. If that is the case, and if the tile t attaches to another tile via its East edge, the signal deactivates the glue g_s, that is, it changes its state to *off*. Signals can change the state of a glue from *latent* to *on* or to *off*, or from *on* to *off*. Note that, once a glue is in the *off* state, its state cannot be changed anymore. A tile can send a signal to its neighbour tile by activating a glue on an edge that is common with a neighbour tile. Signals can change the state of the glues, therefore signals can activate new glues and thus initiate a signal in the next tile. Moreover, signals can activate glues on a free edge and make new attachments possible. In addition to the activation, signals can deactivate the glues and, as a result, an existing structure might become unstable. In the STAM model, if the deactivation of a glue makes a structure unstable, the structure will break apart into two stable components.

Informal Introduction to DTAM

Here we define the *Detachable Tile Assembly Model (DTAM)* model as a variation of STAM. DTAM is a weaker version of STAM, whereby the signals are used only to *deactivate* glues. Thus all glues will start in the state *on*, and the state *latent* is not used. In addition, the signals themselves can only be turned *on*. Note that, since DTAM does not use the signals to activate glues, no new attachment possibilities (besides those that were present in the initial set-up) will be introduced during the self-assembly process. In Section 4 we will show that, in spite of these restrictions, the use of DTAM reduces the tile complexity of the construction of a thin rectangle as compared to [1].

We also introduce the *Simplified Detachable Tile Assembly Model (SDTAM)*, which is a restricted version of SDTAM wherein one starts with a single seed tile and, at each step, only the attachment of a single tile to the current configuration is allowed. In Section 3 we will prove that SDTAM is Turing universal at temperature 1.

2.1 Formal Definition of DTAM

Detachable Tiles

A *detachable tile* is a unit square with the following properties. On each of its edges it has a set of *glues* and a set of *signals*, each of whom can be in a state from $Q = \{on, off\}$. Figure 1 part *(i)* shows an example of a tile in aTAM, and part *(ii)* illustrates the glues and signals of a detachable tile in DTAM. To each tile, we also associate a transition function, as described in the following. Note that, for simplicity, the states of the glues and signals are showed by superscripts: The superscript '+' indicates the state *on* and the superscript '-' indicates the state *off*. Since we only consider self-assembly models with detachable tiles, in the remainder of the paper we will call a *detachable tile* simply a *tile*.

Let Γ (the set of glues) and Σ (the set of signals) be two finite alphabets. The set of directions $D = \{N, E, W, S\}$ is the set of directions North, East, West, and South, respectively. If $d \in D$ is a direction, we define $\bar{d}$ to be the opposite direction of d, where $\bar{W} = E$, $\bar{N} = S$, $\bar{E} = W$, and $\bar{S} = N$.

A *tile* T over the alphabet $\Gamma \times \Sigma$ is a 4-tuple $t = (G_t, S_t, \Delta_t)$ where $G_t : D \to \mathcal{P}(\Gamma \times Q)$ is a function which, for every direction $d \in D$, specifies the set of glues on the edge d of the tile t, together with their respective states. Similarly, $S_t : D \to \mathcal{P}(\Sigma \times Q)$ is a function which, for every direction $d \in D$, specifies the set of all signals on the edge d of the tile t, together with their respective states.

Note that if, for a direction $d \in D$, we have that $G_t(d) = \emptyset$ (respectively $S_t(d) = \emptyset$), this means that there are no glues (respectively no signals) on the edge d of the tile t.

The *transition function* of the tile t is defined as a function $\Delta_t : D \times \Sigma \to \mathcal{P}((D \times \Gamma) \cup (D \times \Sigma))$. In DTAM, glues can only be deactivated, and signals can only be turned *on*, that is, $(d, g) \in \Delta_t(d', s')$ means that an active glue g^+ on the d edge of the tile will be deactivated (become g^-), and $(d, s) \in \Delta_t(d', s')$ means that an *off* signal s^- on the d edge of the tile will be turned *on* (become s^+).

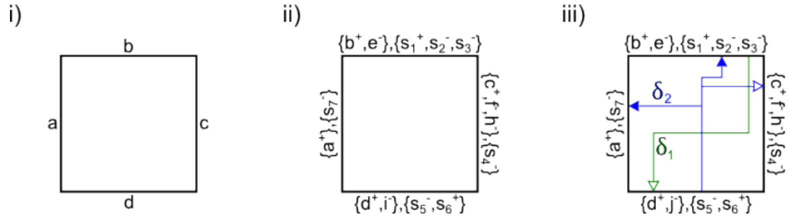

Fig. 1 Part *(i)* shows a tile in the aTAM model: the tile has one glue on each side. Part *(ii)* shows the glues and signals on the edges of a detachable tile in the DTAM model: On the North edge, the presence of the set $\{b^+, e^-\}$ denotes that glue b is *on* and glue e is *off*, while the set $\{s_1^+, s_2^-, s_3^-\}$ denotes that the signal s_1 is *on*, and that signals s_2 and s_3 are *off* at this time. Part *(iii)* shows the tile defined in Example 01. The transition function Δ_t has two transitions. First, the transition $\delta_1 = \Delta_t(N, s_3) = \{(S, j)\}$ (green) starts from the signal s_3 on the North edge, and deactivates the glue j on the South edge. Second, the transition $\delta_2 = \Delta_t(S, s_5) = \{(E, c), (N, s_2), (W, s_7)\}$ (blue) starts from the signal s_5 on South edge, deactivates the glue c on the East edge, turns on the signal s_2 on the North edge, and turns on the signal s_7 on the West edge. The arrows with filled arrowheads represent the paths that turn signals *on*, while the arrows with empty arrowheads represent the paths that signals walk through to deactivate glues. All transitions are considered pending, and will be applied only if the originating signal (s_5, respectively s_3) enters the *on* state.

Note that, for a tile t, the transition function Δ_t is invariant, but for each direction d, the states of glues $G_t(d)$, and the states of signals $S_t(d)$ change over time.

Transitions can only be applied when the corresponding signals are *on*. For example, the glue g^+ on the edge d of the tile t will be deactivated if and only if $(d, g) \in \Delta_t(d', s')$ and $(s', on) \in S_t(d')$. Similarly the signal s^- will be turned on if and only if $(d, s) \in \Delta_t(d', s')$ and $(s', on) \in S_t(d')$.

In addition, the tile t must satisfy the following conditions:

1. There can be only one glue and one signal of each kind on any given edge of a tile. In other words, for a given tile t, for all $d \in D$ and for all $(g, q), (g, q') \in G_t(d)$, we have $q = q'$. Similarly for signals, for a given tile t, for all $d \in D$ and for all $(s, q), (s, q') \in S_t(d)$, we have $q = q'$.
2. Transitions in Δ_t can only deactivate glues that already exist on an edge. This means that, if $(d, g) \in \Delta_t(d', g')$ then $(g, q) \in G_t(d)$ for some $q \in Q$.
3. Transitions in Δ_t can only activate signals that already exist on an edge. This means that, if $(d, s) \in \Delta_t(d', g')$ then $(s, q) \in S_t(d)$ for some $q \in Q$.
4. Transitions in Δ_t cannot deactivate their own starting signals. This means that $(d, s) \notin \Delta_t(d, s)$.

The following example, illustrated in Figure 1 part *(iii)*, describes a detachable tile with the states of its glues, states of its signals, and its transitions. Note that, in this and subsequent figures, transitions are depicted by arrows: The origin (respectively the destination) of the arrow in the tile indicates the signal that must be turned on for the transition to be applied (respectively which signal is turned on by this transition, or which glue is deactivated by this transition). The convention that we use is that arrows depicting transitions which turn signals *on* have filled arrow-heads, while those depicting transitions that deactivate glues have empty arrowheads.

Example 01 *Assume that* $\Gamma = \{a, b, c, e, f, h, i, j\}$ *and* $\Sigma = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$. *The tile* t *is defined as* $t = (G_t, S_t, \Delta_t)$ *where* $G_t(W) = \{a^+\}$, $G_t(N) = \{b^+, e^-\}$, $G_t(E) = \{c^+, f^-, h^-\}$, $G_t(S) = \{d^+, j^-\}$, *and* $S_t(W) = \{{s_7}^-\}$, $S_t(N) = \{{s_1}^+, {s_2}^-, {s_3}^-\}$, $S_t(E) = \{{s_4}^-\}$, $S_t(S) = \{{s_5}^-, {s_6}^+\}$. *The transition function* Δ_t *is defined by* $\Delta_t(N, s_3) = \{(S, j)\}$ *and* $\Delta_t(S, s_5) = \{(E, c), (N, s_2), (W, s_7)\}$. *Informally, the transition* $\delta_1 : \Delta_t(N, s_3) = \{(S, j)\}$ *starts from the signal* s_3 *on the North edge and activates the glue* j *on the South edge. Similarly,* $\delta_2 : \Delta_t(S, s_5) = \{(E, c), (N, s_2), (W, s_7)\}$, *starts from the signal* s_5 *on the South edge and deactivates the glue* c *on the East edge, turns on the signal* s_2 *on the North edge, and turns on the signal* s_7 *on the West edge. The transitions are considered pending, and will only be applied if the corresponding signals enter in the on state.*

2.2 Transitions

Before the formal definition of *transitions* and *DTAM*, we review the definitions of *assembly-graph*, *glue strength*, *configuration*, and *supertile*.

Configurations and Associated Binding Graphs

Let $f\colon A \to B$ be a function and $A' \subseteq A$. The *restriction* of f to A' is a function $f|_{A'}\colon A' \to B$ defined as $f|_{A'}(x) = f(x)$ for all $x \in A'$.

A *pseudo-grid graph* is a directed weighted graph $G = (N_G, E_{Gv} \cup E_{Gh}, \pi_G)$ where N_G is a finite set of nodes, $E_{Gv}, E_{Gh} \subseteq N_G \times N_G$ are two sets of edges (called the vertical and horizontal edges respectively) such that $E_{Gv} \cap E_{Gh} = \emptyset$, and the function $\pi_G : (E_{Gv} \cup E_{Gh}) \to \mathbb{Z}^+$ is a weight function that associates a weight to every edge in the graph. The *node-induced subgraph* of G by the subset $N'_G \subseteq N_G$ is defined as the graph $G|_{N'_G} = (N'_G, {E_G}'_v \cup {E_G}'_h, \pi'_G)$ where ${E_G}'_v = \left\{(\alpha, \beta) \in E_{Gv} \;\middle|\; \alpha, \beta \in N'_G\right\}$ and ${E_G}'_h = \left\{(\alpha, \beta) \in E_{Gh} \;\middle|\; \alpha, \beta \in N'_G\right\}$, while π'_G equals $\pi_G|_{{E_G}'_v \times {E_G}'_h}$.

If Γ is a set of glues, a *glue-strength* function g over Γ is defined as $g : \Gamma \to \mathbb{Z}^+$ where $\mathbb{Z}^+ = \{x \in \mathbb{Z} | x \geq 0\}$, and for a glue $\Gamma \in \Gamma$, $g(\Gamma)$ is called the *glue-strength* of the glue Γ or shortly the *strength* of Γ.

Let Θ be a set of tiles over $\Gamma \times \Sigma$, and let g be a glue-strength function over Γ. A (partial) mapping $C : \mathbb{Z}^2 \to \Theta$ is called a *general configuration* over the tile set Θ.

A pseudo-grid graph $G_C = (N_G, E_{Gv} \cup E_{Gh}, \pi_G)$ is called the *assembly-graph associated to the general configuration* C over the tile set Θ, if there is a bijection $f\colon N_G \to \mathrm{dom}(C)$ such that for all $\alpha, \beta \in N_G$, we have $(\alpha, \beta) \in E_{Gv}$ if and only if $f(\alpha) + (0, 1) = f(\beta)$, and $(\alpha, \beta) \in E_{Gh}$ if and only if $f(\alpha) + (1, 0) = f(\beta)$.

The weight function π_G of G_C is defined as follows. Assume, without loss of generality, that $e = (\alpha, \beta)$ is a vertical edge in E_{Gv}, from node α to node β, where $f(\alpha) = (x_1, y_1)$, and

$C(x_1, y_1) = (G_1, S_1, \Delta_1) \in \Theta$ while $f(\beta) = (x_2, y_2)$, and $C(x_2, y_2) = (G_2, S_2, \Delta_2) \in \Theta$. The weight of an edge e of the assembly-graph G_C is defined as $\pi(e) = \sum g(\Gamma)$ where Γ ranges over all glues that are active on both the North edge of tile $C(x_1, y_1)$ and the South edge of tile $C(x_2, y_2)$. If there is no glue Γ with this property then $\pi(e) = 0$. The weight function is defined similarly for horizontal edges.

In other words, the nodes of an assembly-graph G_C associated to a general configuration C can be embedded in $\mathbb{Z}^2$ such that every edge in E_{Gv} has length 1 and points upwards, every edge in E_{Gh} has length 1 and points rightwards, two nodes with Euclidean distance 1 are connected by an edge, and a tile is placed at each node. Note that graph edges are different from (perpendicular to) the edges of the tiles, with each graph edge corresponding to an attachment between two adjacent tiles, see Figure 2. The weight of a graph edge indicates the power of the attachment of corresponding adjacent tiles.

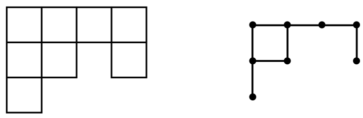

Fig. 2 An example of a general configuration and the assembly-graph associated to it. All the edges of the graph are considered to be directed, with all the vertical edges pointing up, and all the horizontal edges pointing to the right.

The general configuration C over a tile set Θ is called a *connected configuration* or simply a *configuration* or *supertile* if the assembly-graph associated to C is connected. The configuration C is called *stable* at temperature τ (τ-stable supertile), if either

- For all i and $j \in \mathbb{Z}, C(i,j)$ is undefined except for one position (x, y), where $C(x, y) \in \Theta$ or,
- The sum of the weights of the edges each of the the possible cuts of the assembly graph (all the edges between two partitions in the partitioning of the vertices of the assembly graph associated to C into two disjoint sets) associated to C is greater than or equal to τ.

A supertile C over Θ is called a *signal-stable supertile* if none of the transitions Δ_t of tiles t in C are applicable, moreover, only the edges on the borders of the configuration have their signals in the state *on*.

The self-assembly process proceeds by repeated applications of one of the following three types of transitions.

Attachment Transitions

During an attachment transition, two stable shape-compatible supertiles, with sufficient glue-strength at their abutting perimeter, assemble to form a larger supertile as shown in Figure 3 part (i) and (ii).

Formally, we say that the supertile V with associated assembly-graph $G_V = (N_G, E_{Gv} \cup E_{Gh}, \pi_G)$ is the result of an attachment of the two τ-stable supertiles V_1 and V_2, if there exist two sets $P_1 \subset N_G$, $P_2 \subset N_G$, with $P_1 \cup P_2 = N_G$, and $P_1 \cap P_2 = \emptyset$, satisfying the following conditions:

1. $\mathrm{dom}(V) = \mathrm{dom}(V_1) \cup \mathrm{dom}(V_2)$, $\mathrm{dom}(V_1) \cap \mathrm{dom}(V_2) = \emptyset$
2. $V(x, y) = V_1(x, y)$ for all $(x, y) \in \mathrm{dom}(V_1)$, and $V(x, y) = V_2(x, y)$ for all $(x, y) \in \mathrm{dom}(V_2)$, with the exception of any adjacent tiles at positions $(x_1, y_1), (x_2, y_2) \in$

$\mathbb{Z}^2$ where $V_1(x_1, y_1) = t_1 = (G_1, S_1, \Delta_1)$, $V_2(x_2, y_2) = t_2 = (G_2, S_2, \Delta_2)$, $V(x_1, y_1) = t'_1 = (G'_1, S'_1, \Delta'_1)$, $V(x_2, y_2) = t'_2 = (G'_2, S'_2, \Delta'_2)$, where the following conditions hold:
 - The signal s^+ exists on the edge d of the tile t_1 that abuts t_2, that is, $(s, on) \in S_1(d)$,
 - The signal s^- exists on the corresponding edge $\bar{d}$ of the tile t_2, $(s, off) \in S_2(\bar{d})$.

 If these conditions are satisfied, then, the state of the signal s^- on the edge $\bar{d}$ of t_2 is changed by this attachment transition to *on*, that is, $S'_2(\bar{d}) = (S_2(\bar{d}) \setminus \{(s, off)\} \cup \{(s, on)\}$. All other elements of t_1 and t_2 remain unchanged.
3. The supertile V is stable at temperature τ.

Informally, condition (2) stipulates that if, during an attachment transition, two tiles attach and one of them has the signal s^+ on the abutting edge, then the signal s on the corresponding edge of the second tile also becomes *on*.

Detachment Transitions

During a detachment transition, a supertile in which the glue-strength along an internal "cut" is not strong enough (due to the deactivation of one or more glues), breaks into two smaller stable supertiles, as shown in Figure 3 part (v).

Formally, supertiles V_1 and V_2 are the result of a detachment transition applied to a supertile V with associated assembly graph $G_V = (N_G, E_{Gv} \cup E_{Gh}, \pi_G)$, if there exist two sets $N_1, N_2 \subset N_G$, such that $N_1 \cup N_2 = N_G$, $N_1 \cap N_2 = \emptyset$, and the following conditions hold:

1. $\text{dom}(V) = \text{dom}(V_1) \cup \text{dom}(V_2)$, $\text{dom}(V_1) \cap \text{dom}(V_2) = \emptyset$,
2. $V(x, y) = V_1(x, y)$ for all $(x, y) \in \text{dom}(V_1)$, and $V(x, y) = V_2(x, y)$ for all $(x, y) \in \text{dom}(V_2)$,
3. The weight of the cut (N_1, N_2) is smaller than τ, and $G|_{N_1}$ is the assembly-graph associated to V_1, while $G|_{N_2}$ is the assembly-graph associated to V_2.

Note that the supertiles V_1 and V_2 that result from a detachment transition are not necessarily τ-stable, and may further break into smaller supertiles if there exist cuts along which the attachments are not strong enough.

Action Transitions

The result of an action transition applied to the τ-stable supertile U is the supertile V, iff V coincides with U in all positions but one, where the tile in the supertile U appears *before* the application of the action transition, while the tile in the supertile V appears *after* of the application of that action transition. Note that there are two types of actions transitions: those that turn signals *on* (Figure 3 (iii)), and those that deactivate glues (Figure 3, (iv)).

Formally, a supertile V is the result of applying one of the transitions of one of tiles of the τ-stable supertile U, iff for all $(x, y) \in \mathbb{Z}^2, V(x, y) = U(x, y)$ except two adjacent positions $(x_1, y_1), (x_2, y_2) \in \mathbb{Z}^2$ where $U(x_1, y_1) = t_1 = (G_1, S_1, \Delta_1)$, $U(x_2, y_2) = t_2 = (G_2, S_2, \Delta_2)$, $V(x_1, y_1) = t'_1 = (G'_1, S'_1, \Delta'_1)$, and $V(x_2, y_2) = t'_2 =$

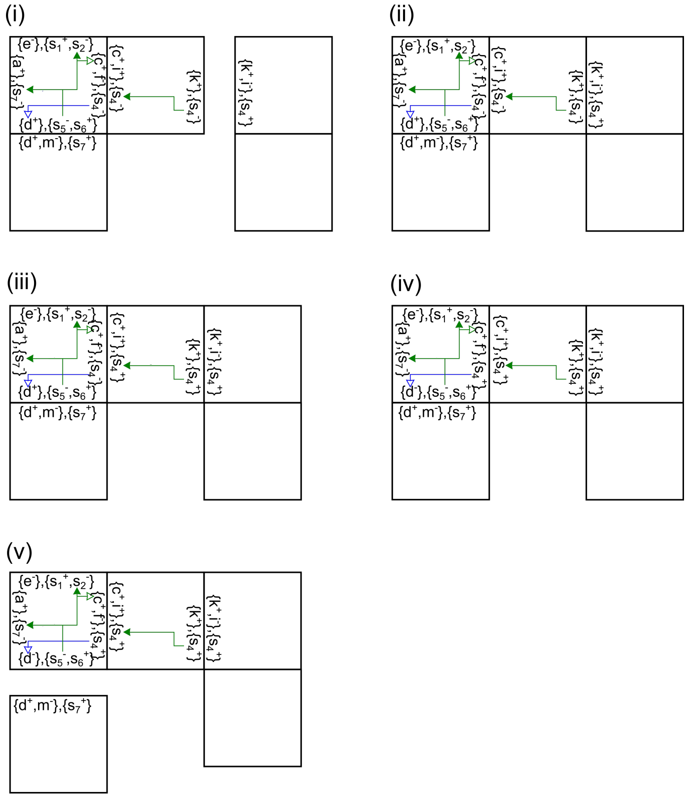

Fig. 3 Parts (i) and (ii) show an example of the attachment transition: Two supertiles attach via the active glue k^+ (one assumes that the strength of the glue k is larger than the temperature). Moreover, since the signal $s_4{}^+$ on the top-right tile of the newly formed supertile is *on*, the signal s_4^- on the abutting edge of its western neighbour tile is turned *on*, becoming s_4^+. Parts (iii) and (iv) illustrate two consecutive action transitions: part (iii) - turning a signal *on*, part (iv) - deactivating a glue. First, since the top-middle tile contains the transition $(s_4, W) \in \Delta_1(E, s_4)$ (illustrated by the green arrow with filled arrowhead), and since the signal s_4 on the East edge of the top-middle tile is *on*, during the first action transition the signal s_4 on the West edge of the top-middle tile is turned *on*, becoming s_4^+. Second, since the top-left tile contains the transition $(d, S) \in \Delta_2(E, s_4)$ (illustrated by the blue arrow with empty arrowhead), and since the signal s_4 on the East edge of the top-left tile is *on*, during this second action transition the glue d on the South edge of the tile is deactivated, becoming d^-. Part (v) illustrates a detachment transition: because there are no more active glues to hold together the top-left tile and the bottom-left tile of the supertile, the latter detaches from the supertile.

(G_2', S_2', Δ_2'), the tiles t_1 and t_2 are adjacent on the direction d of t_1, and one of the following conditions holds:

– (glue deactivation) There exists a glue $g \in \Gamma$ and a signal $s \in \Sigma$ such that $(d', g) \in \Delta_1(d, s)$ and $(s, on) \in S_1(d)$. Then, $S_1 = S_1'$, $\Delta_1 = \Delta_1'$, $t_2 = t_2'$. The sets G_1 and G_1' are the same except that $G_1'(d') = (G_1(d') \setminus \{(g, q) | q \in Q\}) \cup \{(g, \mathit{off})\}$.
– (turning a signal *on*) There exist a signal $s \in \Sigma$ and a signal $s' \in \Sigma$ such that $(d', s') \in \Delta_1(d, s)$ and $(s, on) \in S_1(d)$. Then, $G_1 = G_1'$, $S_1 = S_1'$, $\Delta_1 = \Delta_1'$, $G_2 =$

G'_2, $\Delta_2 = \Delta'_2$. The sets S_2 and S'_2 are the same except that, if $(s', q) \in S_2(\bar{d}')$, for some $q \in Q$, then $S'_2(\bar{d}') = (S_2(\bar{d}') \setminus \{(s', q) | q \in Q\}) \cup \{(s', on)\}$.

A *Detachable Tile Assembly System (DTAM)* over alphabets Γ and Σ is a 5-tuple $(\Theta, g, \tau, \lambda, f)$, where Θ is a set of tiles over $\Gamma \times \Sigma$, g is a strength function over Γ, τ is a positive integer (temperature), λ is a finite set of starting τ-stable configurations, and $t_f \in \Theta$ is a single tile which will "mark" the final configuration of the assembly. The self-assembly process begins with a starting configuration from λ and proceeds by successive applications of the three types of transitions, asynchronously and non-deterministically. We say that a set of configurations $\mathcal{C}$ is *final* if, starting from λ, we can obtain $\mathcal{C}$ by iteratively applying any of the three types of transitions, and, moreover, no other transitions can be applied to $\mathcal{C}$. We call a supertile $Z \in \mathcal{C}$ a *final supertile* if Z is a τ-stable signal-stable supertile, moreover, there exist $i, j \in \mathbb{Z}^2$ such that $Z(i, j) = t_f$.

In comparing DTAM (the model we introduced in this section) with STAM, note first that, while DTAM has sets of signals separate from the sets of glues, these signals could easily be simulated in the STAM model as glues with strength zero. Thus, the introduction of the sets of signals as notation does not enhance the STAM model, that is, DTAM is not a generalization of STAM in this respect. The second observation is that in DTAM glues can only be deactivated and signals can only be activated: In this sense DTAM is a weaker version of STAM. Thirdly, since glues can only be deactivated, and signals can only be activated, the sets of pending actions are not needed in DTAM. Thus, while the formalism is slightly different, the DTAM model is a simplified version of STAM.

A *Simplified DTAM (SDTAM)* is a simplified version of DTAM, where all definitions are the same except those of the seed configuration and attachment transitions: In SDTAM, a seed configuration consist of a single seed tile, and the attachment of two supertiles is restricted to the case where one of the supertiles consists of a single tile.

3 Turing Universality

In this section we show that the SDTAM model can simulate a Turing machine at temperature $\tau = 1$. The basis of the proof is the simulation by SDTAM of any given *deterministic zig-zag tile assembly system* at temperature $\tau = 2$ – a type of self-assembly system that was proved in [4] to be Turing universal. Moreover, our construction utilizes an SDTAM with at most one signal per tile, and where signals travel through only one tile before deactivating a glue.

A *deterministic zig-zag tile assembly system* at temperature $\tau = 2$ is a system that deterministically assembles a structure row by row, growing only upwards: Each row has to be completed before the next row starts, and each row grows in the opposite direction compared to the previous row. Moreover, the width of each row can only be greater than or equal to the width of the preceding row. For example, the first row could start from a seed tile and only grow to the right until a tile is placed that has a North glue of strength 2 and no East glue; then the second row starts assembling above this tile and has to grow to the left. See Figure 4 (i), where the order of tile placement is illustrated in that the directed path shows the direction of the growth of the assembly. Figure 4 (ii) illustrates the way in which attachment happens in such a system: Here, for any pair of tiles (t, t'), a small arrow enters tile t' from tile t if tile t was attached to the structure before tile t', and tile t' attaches to the structure via the edge indicated by the arrow. Thus, incoming arrows in a tile indicate the edges by which the tile attached to the structure, and outgoing arrows indicate the edges by which the subsequent tile will attach. For example, the arrows of the top-right tile of Figure 4 (ii) indicate that the tile attached to the structure through its East and South glues, each of temperature 1, and that the next tile of the assembly will attach to the structure through the North glue of this tile, which has to have temperature 2 since only one edge is used for attachment. Figure 4 (iii) shows the 7 different ways in which tiles can attach in a temperature 2 deterministic zig-zag tile assembly system. We refer to [4] for a more detailed description.

In our construction, for a given deterministic temperature 2 zig-zag tile assembly system $\Phi = \langle T, s, 2\rangle$, where T is a set of aTAM tiles, s is the seed tile, and 2 denotes the temperature, we define an SDTAM self-assembly system at temperature1, namely $\Psi = (\Theta, g, \tau, \lambda, f)$ that simulates Φ. In this construction, each tile $t \in T$ will be replaced with a "gadget" of new tiles in Ψ that encodes the glues of t in the glues and shape of its borders. A "gadget" is a set of new tiles that simulate the behaviour of the original tile by uniquely assembling into a super-tile with the same attachment properties as the originating tile t.

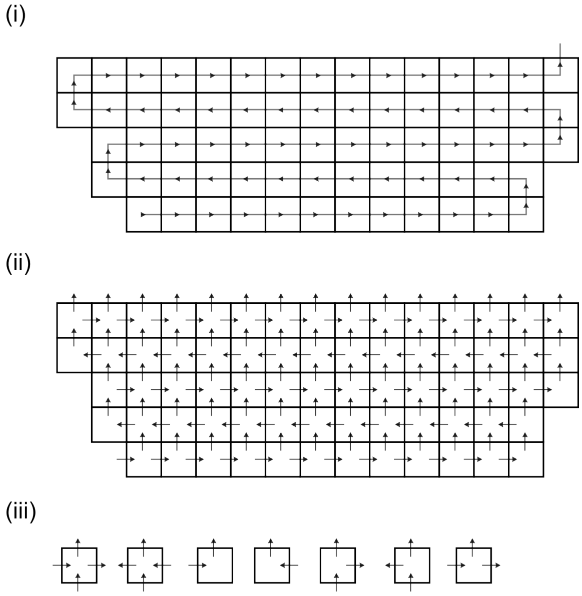

Fig. 4 Simulation of a Turing machine using a deterministic zig-zag tile assembly system at temperature 2. The directed path in part (i) shows the direction of the growth of the assembly. Part (ii) illustrates the way in which attachment happens in such a system: Here, for any pair of tiles (t, t'), a small arrow enters tile t' from tile t if tile t was attached to the structure before tile t' and tile t' attaches to the structure via the edge indicated by the arrow. Part (iii) shows all the possible kinds of tile types in such a system, based on the way they attach to the structure.

We will now simulate a deterministic zig-zag tile assembly model at temperature 2 with an SDTAM at temperature 1. Similar to the construction in [2] and [4], the simulation will be achieved by replacing each original tile by a "gadget" of new tiles. Since the new tile system operates at temperature 1, this construction alone is not sufficient to guarantee the growth of each gadget uniquely, and incorrect attachments can form. Behsaz et al. [2] employed staged tile assembly system to control the assembly growth. In our construction, we will use signals that lead to the detachment of incorrect growths to solve the same problem.

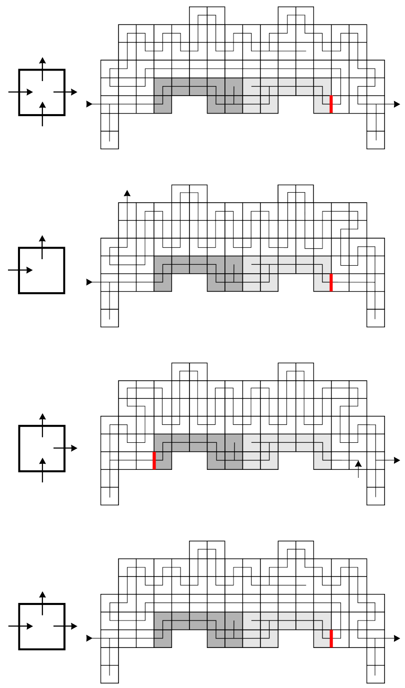

Fig. 5 In each of the two columns, a tile on left is an original tile from the deterministic zig-zag tile assembly model at temperature 2, and its corresponding gadget made out of SDTAM tiles is shown at its immediate right. For example, the top left tile with North glue 00 (right/right) and South glue 10 (left/right) is simulated by the gadget immediately next to it. This gadget has on its North side two bumps both positioned at the right of their respective two-level blocks. The gadget has at its South two dents, the first positioned at the left and the second positioned at the right of their corresponding two-level blocks. The arrows on the gadget show the order of growth of the assembly.

Theorem 01 *For any deterministic temperature 2 zig-zag tile assembly system* $\Gamma = \langle T, s, 2\rangle$ *there exists a simplified detachable tile assembly system SDTAM at temperature* $\tau = 1$ *that simulates* Γ *with horizontal scale* $O(\log(|T|))$.

Proof The proof is by construction. Each tile in the tile set T is replaced by a set of new tiles with the property that they self-organize uniquely into a supertile which we will call a "gadget". Figure 5 shows the original tiles and their corresponding gadgets made out of new tiles. The arrows indicate the path that describes the

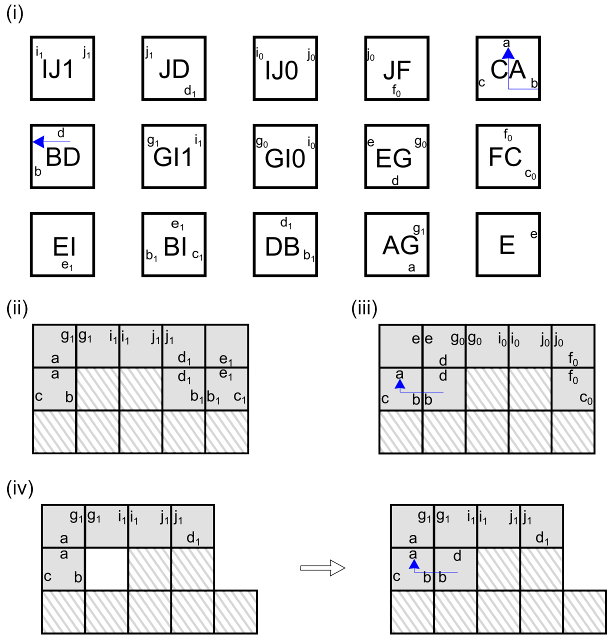

Fig. 6 The (new) tiles in part (i) are used in the bit reader (South glue bit of a gadget) in the construction in Theorem 01. Part (ii) shows the construction of the bit reader when the bump of an incoming bottom gadget indicates bit 1. Part (iii) shows the construction of the bit reader when the bump of an incoming bottom gadget indicates bit 0. Part (iv) shows an example of an incorrect growth, and how it will be fixed. As shown in part(iv), incorrect attachments cannot grow beyond the bump and will stop. Afterward, tile BD can attach to the east edge of the tile CA to detach the incorrect attachments. For readability, in parts (ii), (iii), and (iv), the tile labels have been omitted.

order in which the self-assembly of the gadget proceeds, with the observation that the two bottom "legs" of each gadget self-assemble independently of this path.

Consider for example the original tile and its corresponding gadget situated at the top-left of Figure 5.

The East and West glues of the original tile are simulated as follows. The West glue of the original tile becomes the West glue of the first (new) tile of the path in the corresponding gadget, indicated by the arrow that enters the gadget, see Figure 5, top-left. Similarly, the East glue of the original tile becomes the East glue of the (new) tile of the corresponding gadget indicated by the arrow that exits the gadget.

Since the new tile system is a temperature 1, the fact that sometimes two glue matching sides are needed for an attachment cannot be simulated directly, and a combination of bumps-and-dents will be needed to create the same effect. To that end, the North and South glues of the original tile are simulated as follows.

The glues on the North and South edges of t are first encoded as binary numbers. Since there are no more than $4 * |T|$ glues, we only need to use numbers from zero to $4 * |T|$ for the coding, and only $\log(4 * |T|)$ bits are needed to encode these numbers. To simulate the North glue of the tile t, each bit in the binary number encoding that glue is encoded as a *two-level block with a bump*, consisting of seven new tiles (5 tiles in the bottom row and 2 tiles in the top row). If the two tiles in the top row are located on top of the second and third tile of the bottom row, that is, the bump is positioned at the left, then the bit is 1 (see Figure 6 (ii), the hashed tiles). Similarly, if the two tiles in the top row are placed on top of the third and forth tile from the bottom row, that is, the bump is positioned at the right, then the bit is 0 (see Figure 6 (iii), the hashed tiles).

Similarly, the glue on the South edge of t is encoded as a binary number implemented by a sequence of two-level blocks with dents (that will geometrically fit into blocks with matching bumps), as follows. Each bit in the binary number representing the South glue of t is encoded by a *two-level block with a dent*, consisting of eight tiles (5 tiles in the top row and 3 tiles in the bottom row). If the three tiles in the bottom row are located under the first, forth, and fifth tile in the top row, that is, the dent is positioned at the left, then the bit is 1 (see Figure 6 (ii), the grey tiles). Similarly, if the three tiles in the bottom row are placed under the first, second and fifth tiles from the top row, that is, if the dent is positioned at the right, then the bit is 0 (see Figure 6 (iii), the grey tiles).

Figure 5 illustrates some tiles and the corresponding gadgets made out of new tiles, that simulate them. For example, the top-left tile with North glue 00 and South glue 10 is simulated by the gadget at its immediate right.

Note that, in each gadget in Figure 5, the (dark and light) grey portions implement the portions that encode the South glue. These portions acts as "bit-readers" in the sense that they "read" the bits that encode the North glue of a gadget that could attach to it from the South. Figure 6 (i) shows the (new) tile types used to encode the bit readers (i.e. South glue). Note that if a tile edge has no label, this means there is no glue on that edge and no attachment can form. The tile can still attach to a structure, via another edge, and glue mismatches are allowed.

Figure 6 (ii) shows bit 1 on the North part of a gadget (hashed) and its corresponding bit reader, i.e. the South glue of a gadget that will fit on top of it (grey). Assuming that the bottom gadget with its hashed portion representing bit 1 is already assembled, the only tile that can attach to it is the one labelled CA, through glue b. After that, the tile labelled AG is the only possible attachment, followed by the attachments of tiles labelled GI1, IJ1, JD, DB, BI and EI, in this order. Similarly Figure 6(iii) shows bit 0 (hashed) of a gadget, and its bit reader, i.e., South glue of a gadget that will fit on top (grey). Assuming that the bottom gadget with its hashed portion representing bit 0 is already assembled, the self-assembly proceeds with BD, CA, EG, E, GI0, IJ0, JF, FC.

Note here that if the West glue of the original tile is labelled c, that glue is transmitted to the bit reader of the corresponding gadget. After all the bits of the South glue of the gadget are formed, the last tile of the bit reader contains in its East glue (marked in red in Figure 5, top-left gadget) information about both the West glue of the original tile, and about its South glue. This information is encoded in the label of the glue, which is c_1 if the West glue was c and the South glue was 1, and c_0 if the West glue was c and the South glue was 0 (see Figure 6(i)).

Incorrect attachments during the assembly of the bit reader (South glue) may form, due to the fact that there may exist original tiles that have the same West glue but different South glues (bit readers). Figure 6(iv) shows how an incorrect attachment may begin to form, and how it is corrected by the use of signals which will detach the incorrect growth. For example, assume we are reading bit 0 (Figure 6(iii)), but the bit reader attempts to read it as a 1 (Figure 6(ii)). After the attachment of tile CA, both tile AG and BD can attach to the current configuration. If, instead of tile BD (which would proceed to correctly read bit 0), tile AG incorrectly attaches, then tiles GI1, IJ1, and JD will attach to the corresponding positions. After the attachment of the tile JD, since the bump representing bit 1 occupies the South side of JD, no further attachment to JD is possible, and the growth of the structure in this direction stops. In order to repair this incorrect growth, when tile BD attaches to the hole on East side of the tile CA, it activates the signal on the east edge of CA. This signal deactivates the glue a on the north edge of the tile CA and detaches the incorrect growth, which can now be replaced by the bit reader for bit 0.

Due to the fact that the zig-zag self-assembly systems that we are simulating is deterministic, once the West glue and the South glue of a tile are known, the other two glues are uniquely determined, so the rest of the white tiles in the gadget in Figure 5 top-left can be hard-coded (this means that the tile at each position is unique and has glues that make sure that the tile can only attach at that particular position).

One can similarly construct gadgets simulating all the other tile types of the given deterministic zig-zag tile system at temperature 2. From the construction it follows that the function that maps each original tile to a set of new tiles that self-organize into the corresponding gadget is a one-to-one correspondence. Since deterministic zig-zag tile systems at temperature 2 are Turing universal, it follows that SDTAM is also Turing universal.

Regarding the scale of the construction, assume that the original tile set has $|T|$ tile types and their glues which, as seen previously, each necessitates $\log(4*|T|)$ bits to encode. Each of the original tiles is simulated by a gadget. The width of such a gadget consists of 3 extra tiles on each side, plus 5 tiles for each bit, for a total of $(3+5*\log(4*|T|)+3)$ tiles. Thus, if the width of a structure assembled in the original deterministic zig-zag tile assembly system at temperature 2 is W, then the width of the structure assembled by SDTAM tiles at temperature 1 that simulates it is $(3+5*\log(4*|T|)+3)*W$, which is $O(\log(|T|)$.

If we ignore the right and left leg of each gadget, and the North bumps (which will interlock with the gadget above it and with the one underneath it, and thus do not add to the height), then the height of a simulating gadget is 5 tiles. If the height of the original zig-zag structure at temperature 2 was H, then the height of the new structure at temperature 1 that simulates it will be $(2+5*H+1)$ (the constants account for the height of the legs of the gadgets on the first row of the new structure, and the height of the North bumps of the gadgets on its top-most row, respectively).

From the construction above it also follows that, besides being Turing universal, this SDTAM utilizes at most one signal per tile, and signals travel through only one tile before deactivating a glue.

□

4 Self-assembly of Thin Rectangles

In this section, we present a DTAM tile assembly system which, for a given $N > 6$, assembles an $N \times N!$ rectangle, and uses only $\mathcal{O}(\log N)$ tile types. Throughout the proof we assume that the DTAM self-assembly system that we construct is at temperature τ, and the power of each glue is either $\tau/2$ or $\tau/3$.

4.1 Informal description of the DTAM tile assembly system

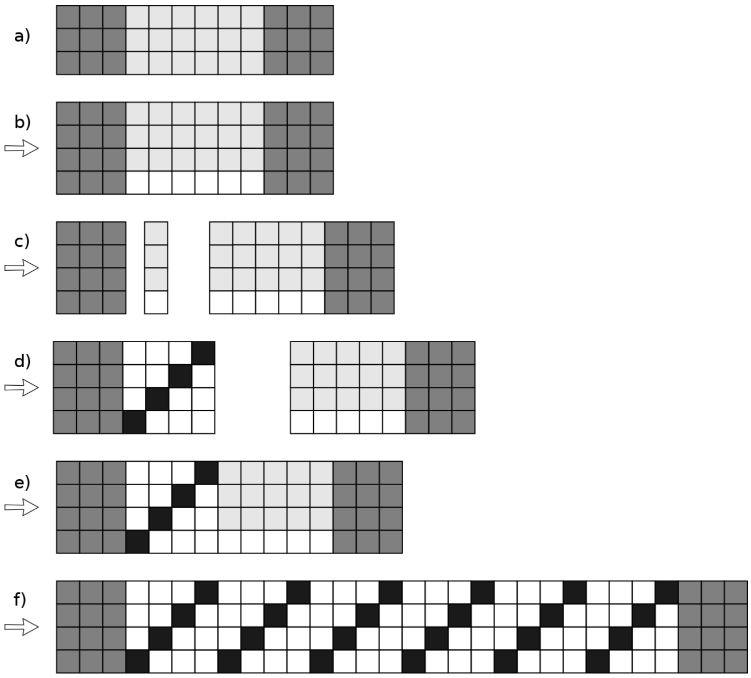

Fig. 7 This figure illustrates the idea of the steps that replace an $M \times (M! + 2C)$ rectangle (7a) by an $(M+1) \times ((M+1)! + 2C)$ rectangle (7f), for $M = 3$ and $C = 3$. (7b) illustrates the addition of a new row to the rectangle in (7a). (7c) illustrates the detachment of the leftmost light grey column. (7d) illustrates the replacement of one light grey column by $(M+1)$ columns of the same shape and size. (7e) illustrates the reattachment of the left part of (7d) to the right part. Repeating steps (7b) to (7e) for all the $M!$ light grey columns results into an $(M+1) \times ((M+1)! + 2C)$ rectangle that is shown in (7f).

We start with a high level explanation of our construction. Figure 7 shows the method that can be used to build an $(M+1) \times ((M+1)! + 2C)$ rectangle starting from an $M \times (M! + 2C)$ rectangle, for a given $M > 6$. This method can then be used for the construction of an $N \times N!$ rectangle as follows. Start with a $6 \times (6! + 2C)$ rectangle, where $C = \lceil \log (N-5) + 2 \rceil$, and use the method (illustrated in Figure 7) to add a row and an appropriate number of columns. C is the width of each of the two dark grey borders of the rectangle in Figure (7a), which control the assembly process but will not be retained in the final structure. Repeat the steps illustrated in Figure (7b)-(7e) for increasing values of M, until an $N \times (N! + 2C)$ rectangle is obtained. At the end, remove the C columns from the left and C columns from the right to obtain the desired $N \times N!$ rectangle.

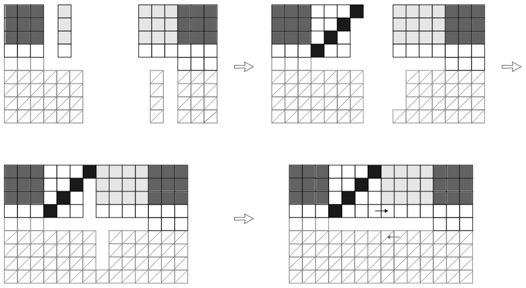

Fig. 8 This figure illustrates the use of the mirror rectangle (hashed) in the process detailed in Figure (7c) through (7e). The mirror rectangle is used to make sure that only the corresponding right and left parts in Figure (7d) can attach to each other (not pieces of rectangles from different steps), and prevent any incorrect attachments by imposing additional geometrical constraints on the attachment.

Figure 7 illustrates the method used to obtain an $(M+1) \times ((M+1)! + 2C)$ rectangle from an $M \times (M!+2C)$ rectangle for the case $M = 3$ and $C = 3$. Note that this is an illustration of the general idea of the method, and not a real example, for which N (and M) should be greater than 6.

The input is a rectangle of size $M \times (M! + 2C)$, see Figure (7a).

Step 1. One row is added to the bottom of this rectangle, as illustrated in Figure (7b).
Step 2. The leftmost white tile of the rectangle in Figure (7b) sends a signal to all the tiles above it, resulting in the detachment of this light grey column from the rest of the rectangle. As a result, the rectangle is now divided into three parts as seen in Figure (7c). (The middle column that was detached in this step is waste, and will not be used anymore.)
Step 3. The remaining left part of the rectangle (its left dark grey portion) expands by $(M + 1)$ columns, Figure (7d).
Step 4. The expanded part (the left part of Figure (7d)), and the right part of Figure (7c) reattach to each other, see Figure (7e).
Step 5. When the two parts reattach, the tile that was the bottom-right tile of the left part of Figure (7d) sends a signal eastwards, to start the detachment of the next light grey column.
Step 6. The above four steps are repeated for all the light grey columns in Figure (7c), resulting in Figure (7f).

The output is a new rectangle of size $(M + 1) \times ((M + 1)! + 2C)$. In the case of our example the output is the $4 \times (4! + 2 \cdot 3)$ rectangle in Figure (7f).

In order to prevent incorrect attachments in the process illustrated in Figure 7, we will need another rectangle, of the same dimensions (see Figure 8, the hashed rectangle at the bottom of each subfigure). This second rectangle is built using a functionally equivalent but disjoint tile set. This means that, if the tile set Θ was used to built the first rectangle as well as the row underneath it, then a mirror tile-set set Θ' can be used, whose tiles are obtained by flipping each of the tiles from Θ about their top-left/bottom-right diagonal but using, instead of its original glues, the corresponding glues from a glue set that is isomorphic to the set of glues of the tiles in Θ. As a result, the tiles from Θ' will self-assemble into a copy of the first rectangle, flipped about its top-left/bottom-right diagonal. We will call this the “mirror rectangle”. The tiles from Θ' will not interact with tiles from Θ. (There are some exceptions, namely some glues that are common to Θ and Θ', that will be explained later in this section.)

The mirror rectangle attaches to the one we wish to expand (Figure (7a)) at the end of the step in Figure (7b), and will stay connected to it for the duration of the expansion. The mirror rectangle expands alongside with the original one, as it has mirror tiles that can self-assemble in the same way.

The purpose of the mirror rectangle is to make sure that in the step from Figure (7d) to Figure (7e), only the left part and the right part of the rectangle in Figure (7d) reattach to each other, and not other partial rectangles.

When all the columns are expanded, the top rectangle and its mirror rectangle detach from each other.

After adding $(N-6)$ rows, the dark grey parts (which encode a counter) stop the self-assembly. Finally, the dark grey columns detach, and only the desired $N \times N!$ rectangle remains.

4.2 Detailed description of the DTAM tile assembly system

Recall that the proposed DTAM self-assembly system is at temperature τ, and the power of the glues is $\tau/2$ or $\tau/3$.

Step 1 (Figure 9 (i), (ii), (iii), (iv)): Addition of a new bottom row, and of the mirror rectangle.

The rectangle we start from consists of C columns on the right side, and C columns on the left side, in addition to the $M!$ columns in the middle. In addition, the rectangle must satisfy the following properties:

- The glues on the south edge of the rectangle follow the pattern shown in Figure 10. (Detailed definitions of the tile types and their glues are shown in Figure 11.)
- The tiles in the white area are connected to their east and west neighbours only with the glues from the set $\{d, d_1, d_2, x, m_1, m_2\}$. (Detailed definitions of the tile types and their glues are shown in Figure 11.)

To this thin rectangle, a new row will be attached at the bottom, as described below.

The tiles from the tile set $\Theta_1 = \{A, B, C, E, F_0, F_1, F_2, F_3, F_4, G, G', H, I, J, K\}$, shown in the first three rows of Figure 11, attach a new row to the bottom of the rectangle, as well as an extra tile (C) underneath it, that connects it to the mirror rectangle. In addition, tiles J and K fill in the positions at the right of tile C (see Figure 9(iv)), as follows.

The tiles C and B are the ones that enable the mirror rectangle to attach under the original rectangle. Their glues b' and c' are mapped to the glues c' and b', respectively, of mirror tiles in the mirror tile set. We define the power of these two glues to be $\tau/2$. As a consequence, the mirror rectangle can attach to the original one using only tiles B and C, through their two glues c' and b', as $\tau/2 + \tau/2 = \tau$ (see Figure 9(iii)).

In order to prevent the detachment of the mirror rectangle from the original one in the next step (Step 2 - column detachment), this connection has to be

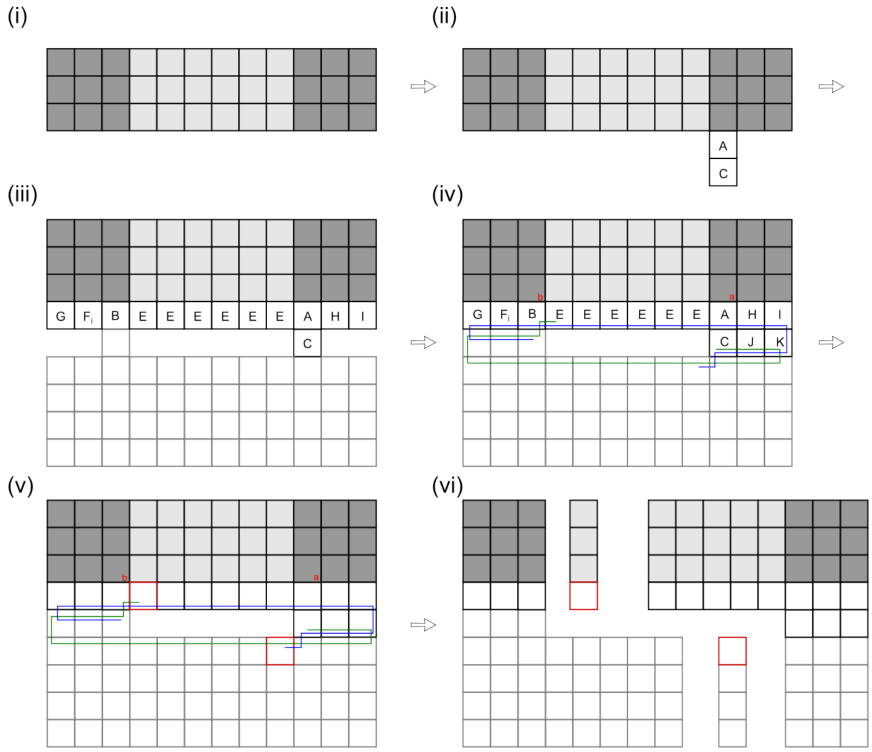

Fig. 9 Illustration of Step 1 and Step 2. (i) is the input rectangle, (ii) shows the attachment of tiles A and C, (iii) shows the addition of the new bottom row and the mirror rectangle, (iv) shows the paths of the detachment signal that is initialized by tile C (green), and its mirror signal path (blue), (v) shows the tiles that are the final recipients of the detachment signals (outlined in red), and (vi) shows the detachment of one column from the input rectangle and of the mirror column from the mirror rectangle.

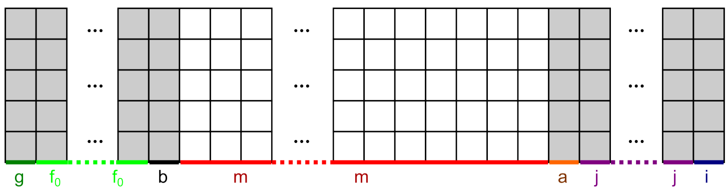

Fig. 10 A thin rectangle that is the input of Step 1 of the construction, and the glues on the tiles of its bottom edge. (Detailed definitions of the tile types and their glues are in Figure 11)

stronger than τ. This is achieved by adding two tile types, J and K, with north glues of strength $\tau/3$. Tiles J and K attach to the structure after the mirror rectangle is completely attached. Their addition makes the connection between the right part of the original rectangle and the mirror rectangle have total strength $\tau/2 + (C-2)\tau/3 + \tau/3 > \tau$ (through tile C, followed by $C-2$ copies of tile J, followed by tile K), see Figure 9(iv). Due to the construction of the mirror tile set tile, mirror tiles of the tiles J and K exist, that will ensure that the connection

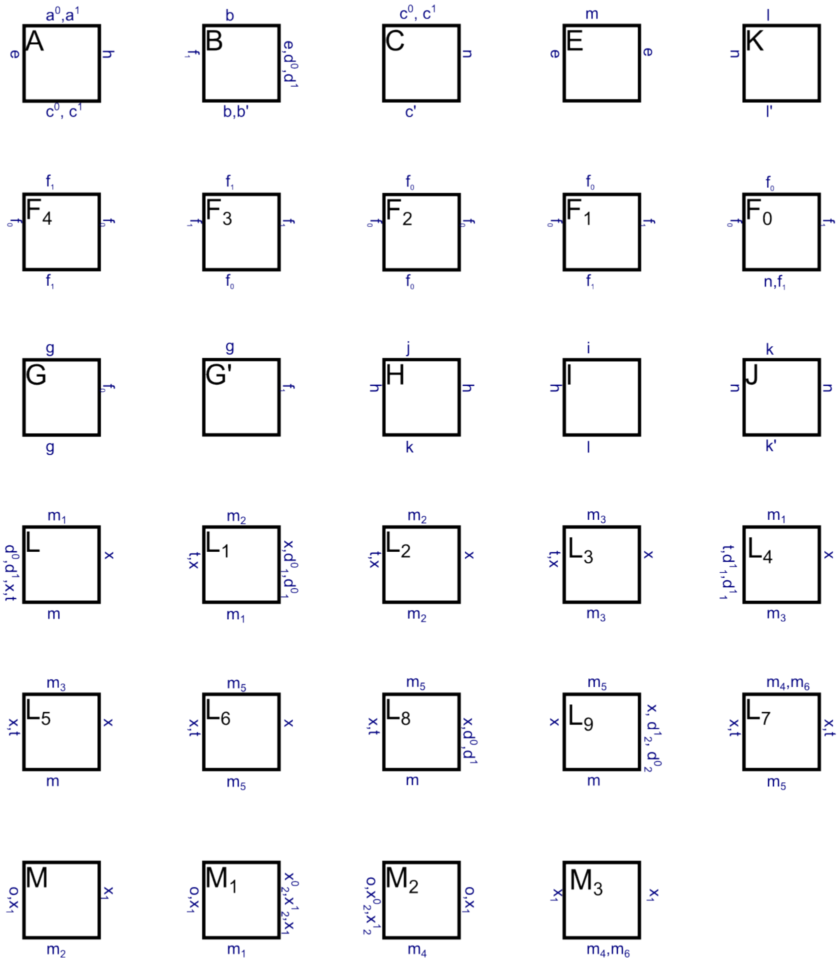

Fig. 11 The tile types that are used in the construction of a thin rectangle and their respective glues (signals are not shown in this figure). The tile types in the top three rows are used in Step 1, and the tile types in the bottom three rows are used in Step 3. The power of all the glues is $\tau/2$, where τ is the temperature of the DTAM system, except for the glues t, k, k', l and l' whose power is $\tau/3$. (Note that glue superscripts 0 and 1 are only used in this figure, to simplify the explanation for the glue strengths. That is, the presence of d^0 and d^1 on the same edge only indicates that glue d is present, and that its strength is the sum the individual strengths of its components d^0 and d^1, both of which have strength $\tau/2$). Functionally, glues x, o, t, d, d_2, d_1 are used for the attachment of columns, while glue m with different subscripts is used for the attachment of rows. Glue f with different subscripts is used for the attachment of the counter. Table 1 and Table 2 include the detailed definition of the signals and transitions for all these tile types.

between the left side of the original rectangle and the mirror rectangle becomes also $\tau/2 + (C-2)\tau/3 + \tau/3 > \tau$.

Note that each of these bonds has a total strength greater than τ. Thus, when the entire structure breaks into two (a left part and a right part) during the next step, the top piece (from the original rectangle) and the bottom piece (from the mirror rectangle, directly underneath it) in each of these parts remain attached together.

Step 2 (Figure 9 (iv), (v), (vi)): Column detachment.

The column detachment is initiated by tile C. Signal s_1 from tile C (shown in green in Figure 9(iv)) travels through tiles J and K, continues through the top row of the mirror rectangle, and then travels through the tiles under G, F_1 and B (the mirror tiles of C, J and K), reaching tile E (marked in red in Figure 9(v)). Similarly, the mirror tile of C (the tile under B) sends the signal s_2 (shown in blue in Figure 9(iv)), which travels a symmetric path and reaches the mirror tile of E (marked in red in Figure 9(v)). If these two signals reach their destination, this guarantees that the two portions that connect the original rectangle with the mirror rectangle are filled in, with no holes. This further implies that the subsequent column detachment will not result, inadvertently, in the separation of any part of the original rectangle from its corresponding part in the mirror rectangle (the total strength of the attachments that keep them together is larger than τ).

The column detachment is illustrated schematically in Figure 9(v) and (vi). When the detachment signal reaches tile E, this tile deactivates the glue on its west edge. Moreover, tile E sends a signal to its north edge, which travels northwards. As a result, all the tiles above tile E, in the same column, deactivate their attachment with their west tile. In order to deactivate the east side of the column, tile E sends another signal to the tile on its east side. This signal goes through all the tiles above it, and deactivates their west attachments. As a result of this process, the column becomes detached from the original rectangle and becomes waste. A similar process, which starts when the signal reaches the mirror tile of E, happens in the mirror rectangle. As a result, a mirror column becomes detached from the mirror rectangle, as seen in Figure 9 (vi).

The details of the process are illustrated in Figure 12. In order to prevent undesired attachments, instead of first deactivating all the west glues of the column tiles, and then deactivating of all its east glues, we alternate. That is, starting with the bottom column tile E, we first detach the west glue of E, then its east glue, as well as send a signal to the tile east of E, deactivating its west glue. As tile E detaches and leaves a hole, the hole is immediately filled by a new tile, L. This new tile is part of the first new column generated during the expansion process of Step 3, and this new tile sends a signal to the tile above it, so as to continue the column detachment. Combining column detachment with the attachment of the first column of the subsequent expansion step, together with the signals that deactivate the glues of the tiles situated east of the column being detached, guarantee that no incorrect attachments can occur. This is because, during the expansion process, when the right and the left parts of the rectangle are separated, the left part will not have any east glue that matches an active west glue from the right part and that could have potentially led to an incorrect attachment.

Step 3 & 4 (Figure 13 and 14): Expansion and re-attachment.

During this step, the left part of the rectangle (as well as the mirror rectangle) is expanded by $(M + 1)$ columns. Figure 11 (last three rows) shows all the tile types that are needed to expand a rectangle with new columns.

Fig. 12 A detailed view of Step 2, and the attachment of the first column in Step 3, that were illustrated schematically in Figure 9(iv): (i) - tile L_8 initiates a signal to detach tile E at the bottom of the first light grey column (marked in red in Figure 9(iv)); (ii) - tile E is detached; (iii) - tile L attaches in the place of tile E and sends a signal to the tile above it, deactivating its west glue; (iv) - all the tiles above tile L are detached and are replaced by tiles from the first column of Step 3; (v) - the left and right part of the rectangle(s) detach from each other; (vi) the first column of Step 3 is completed; (vii) enlarged copy of the bottom rows of part (iii).

The construction started already in Step 2 by the attachment to the left part of the rectangle of tile L, via its west edge (this tile replaces tile E). Subsequently, a square will self-assemble that will attach to the east of the left part of the rectangle (see Figure 13 (ii)). Since the rectangle (that now includes the added bottom row) has height $(M+1)$, the attachment of a square shape amounts to adding $(M+1)$ columns, as required.

The construction of this square is shown in Figure 13 and 14. Starting from tile L, the square self-assembles column by column, using the tile types and glues shown in Figure 13 (i). The last column of the square has tile M_2 in the top row.

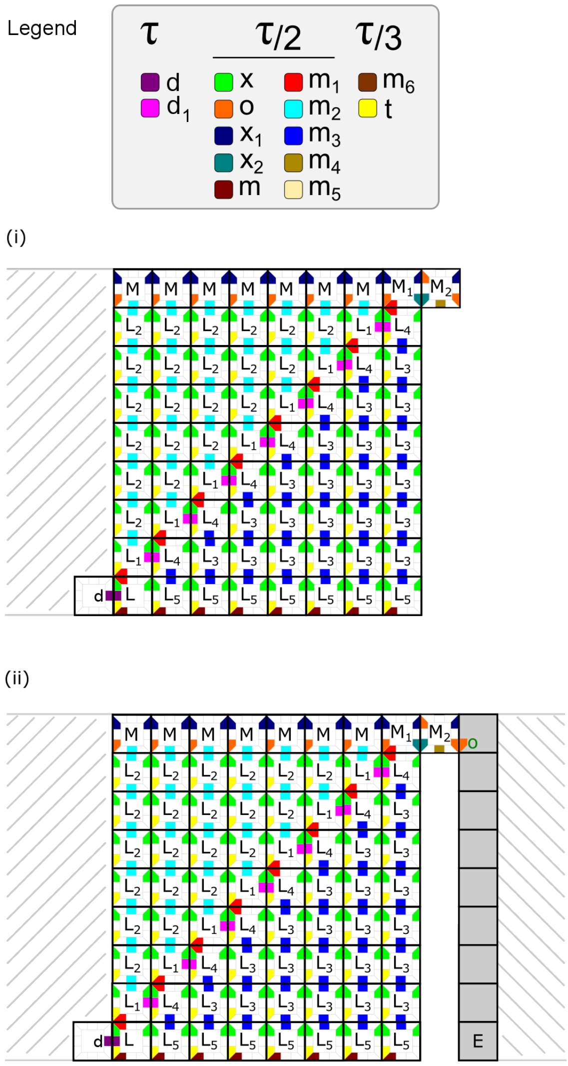

Fig. 13 Together with the next figure (Figure 14), this figure illustrates Step 3 (expansion), and Step 4 (reattachment). The top of the figure lists the colour coding of the glues of tiles involved in these steps, and their strength - the actual glues were listed in Figure 11. (i) shows the construction of the square that starts from tile L (bottom left) and adds the columns of the square one by one, from left to right, ending in tile M_2 (top right); (ii) shows the reattachment of the right part of the rectangle to its left part, through tile M_2 and its mirror tile in the mirror rectangle (the latter is not shown here).

Tile M_2 and its mirror tile in the tile set Θ' bring the corresponding left and right parts of the rectangles together via the glue o (Figure 13 part (ii)). Tiles L_7, L_6, and L_8 complete this last column, filling the gaps between the corresponding left and right parts of the rectangles (see Figure 14 part (i)), and thus reattaching the left and right parts of the rectangles to each other.

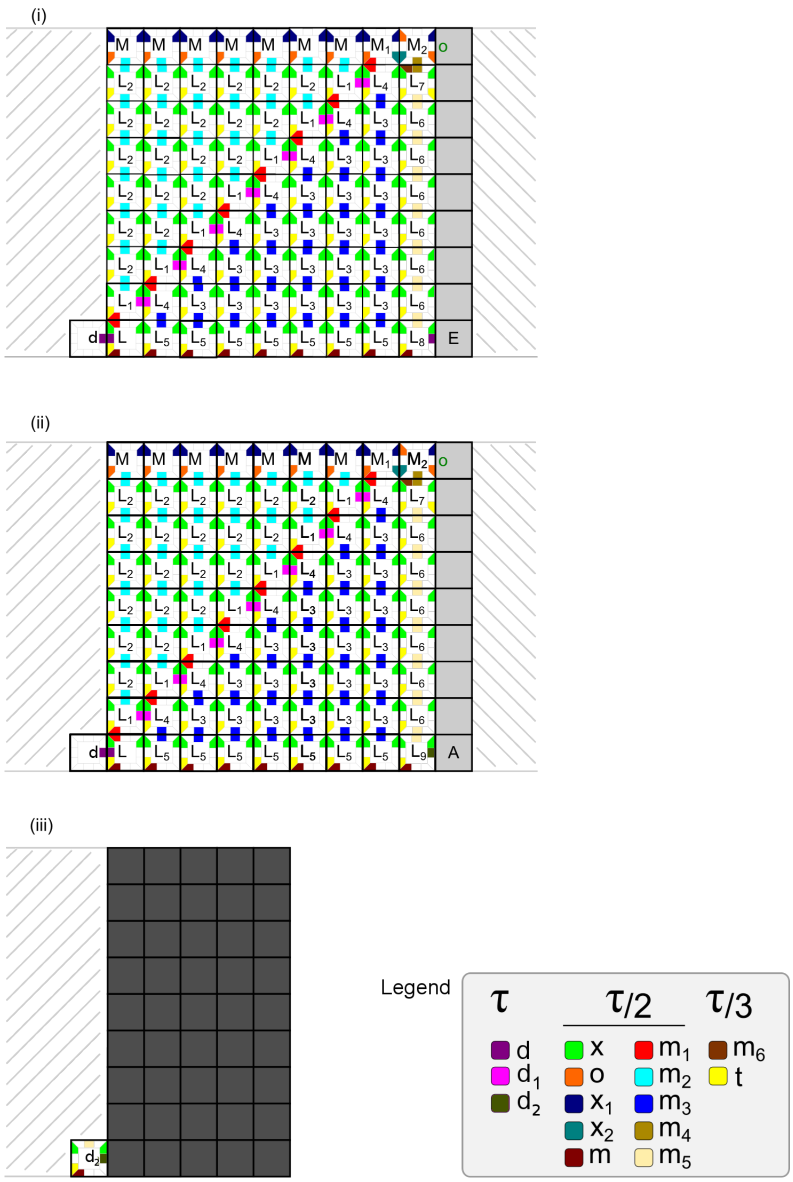

Fig. 14 Together with the previous figure (Figure 13), this figure illustrates Step 3 (expansion), and Step 4 (reattachment).(i) The column underneath tile M_2 is completed, using tiles L_7, L_6 and L_8. This refers to the case when the bottom left tile of the right part of the rectangle is tile E, which signifies that there are still columns to be expanded. (ii) illustrates the case when the bottom left tile of the right part of the rectangle is tile A, which signifies the end of the expansion process. In this case, tile L_9 attaches instead of tile L_8 as the last tile of the column headed by M_2, and the process is continued by the construction of a new dark grey border of the rectangle, as illustrated in (iii).

Step 5 - Initiate the repetition of Steps 2-4 above.

The last tile of the square constructed in Step 3, tile L_8, sends a signal to the tile situated on its east side, which initiates Step 6 - the repetition of Steps 2-4.

Note that the replacement of a column with a square in Steps 3 & 4 is different for the very last column. In this case, the last tile of the square is, instead of tile L_8, a special tile L_9. This tile could not attach previously, because tile E, its east neighbour, does not attach to it, and can only attach when all tiles E have been consumed, and only tile A remains on the same row (see Figure 9 (iv), Figure 14(ii)). Tile A attaches to tile L_9, which sends a signal to tile A which travels through the dark grey part and detaches all of it. Instead of starting a square construction, tile L_9 starts to build a new dark grey portion and the two rows underneath it (see Figure 9(v), (vi) and Figure 14(iii)). The tiles that are required to build this new "border" are hard-coded, and are not included in the tile set presented in Figure 11. The construction of the new border needs $O(C)$ tiles, and the construction of both dark grey borders is the only part of the construction that needs more than a constant number of tiles.

As a result of Steps 1-6, all $M!$ columns, except the dark grey columns, are replaced by $(M+1)\times(M+1)$ squares. Thus, the new rectangle has width $M!\times(M+1)+2C=(M+1)!+2C$.

The procedure above, which started with a rectangle of size $6\times(6!+2C)$, where $C=\lceil log(N-5)+2\rceil$, is then repeated $N-6$ times.

To initiate the termination of the self-assembly, the left dark grey border is used. The left dark grey border acts as a counter, as follows. In order to count the number of rows and stop at N, the tiles F_1, F_2, F_3 and F_4 are used in the standard way, see, e.g., [14][3].

When the counter tile reaches N and stops counting, tile G' attaches to the leftmost column (instead of tile G), at the beginning of Step 1, see Figure 9(iii). Tile G' does not pass the column-detachment signal from its south edge to its east edge and instead will stop the growth process, as follows. Tile G' initiates a signal to deactivate all the glues on the south edges of the last row. As a result, the rectangle above this last row detaches, and cannot grow anymore. This rectangle has size $N\times N!$.

The complete list of tile types in Figure 11, together with their signals can be found in Table 1 and Table 2. Note that the signal s_t, not shown in any of the tiles, starts from tile G' - which replaces tile G in Figure 9(iii) during the last application of Step 6. It then travels towards the east of tile G': If a tile has an active glue s_t on its west edge, this signal activates s_t on its east edge. In addition, for all tiles it encounters, s_t deactivates all glues from the south edge, and thus detaches the mirror rectangle from the output rectangle. The signal s_t is common to all tiles, and is not shown in this table, for readability reasons. Signal s_1 is activated by tile C; the path that it travels is shown in green in Figure 9(iv). Once it reaches tile B, signal s_1 is converted to signal s_5. Tile E changes signal s_5 to s_6 (shown in black, respectively orange in Figure 12(i)), and s_6 starts the column detachment process. Signals s_7, s_8 (red), and s_9 (green), s_{10} (blue) in Figure 12(iii) and Figure 12(vii), are all used in column detachment, as follows. Signals s_7 and s_8 are used to travel the tiles above L. Signal s_9 deactivates the west glues of a column tile, while signal s_{10} deactivates the west edge of its neighbouring east tile, which belongs to the right part of the rectangle. Similarly, signal s_2 is the signal activated by the tile that mirrors C; the path it travels is shown in blue in Figure 9(iv), and it will lead to the column detachment in the mirror rectangle.

Tile Name	List of Signals				Transitions
	North	West	South	East	
A		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
B			s_1^-, s_2^-	s_5^-, s_2^-	$(s_2, W) \to \{(s_2, E)\}$, $(s_1, S) \to \{(s_5, E)\}$
C			s_2^-	s_1^+, s_2^-	$(s_2, E) \to \{(s_2, S)\}$
E		s_2^-, s_6^-, s_5^-		s_2^-, s_6^-	$(s_2, W) \to \{(s_2, E)\}$, $(s_5, W) \to \{(s_6, E), (e, W), (s_1, E)\}$, $(s_6, W) \to \{(e, W)\}$
F_0		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
F_1		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
F_2		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
F_3		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
F_4		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
G			s_2^-	s_2^-	$(s_2, S) \to \{(s_2, E)\}$
G'					s_t^+
H		s_2^-		s_2^-	$(s_2, W) \to \{(s_2, E)\}$
I		s_2^-	s_2^-		$(s_2, W) \to \{(s_2, S)\}$
J		s_1^-, s_2^-		s_1^-, s_2^-	$(s_1, W) \to \{(s_1, E)\}$ $(s_2, E) \to \{(s_2, W)\}$
K	s_2^-	s_1^-, s_2^-	s_1^-		$(s_1, W) \to \{(s_1, S)\}$ $(s_2, N) \to \{(s_2, W)\}$
L	s_8^-, s_7^+, s_9^-	s_5^-	s_7^-, s_8^-, s_9^-	s_{10}^-	$(s_5, W) \to \{(s_7, N)\}$, $(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W), (d, W)\}$
L_1	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-, s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_2	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-, s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_3	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-, s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_4	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-, s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W), (d_1, W)\}$

Table 1 Together with Table 2, this table list the signals and transitions for all the tiles types, except the dark grey border areas. Signals are denoted by the letter s, with subscripts or superscripts, while the other letters indicate glues. The signal s_t, not shown in any of the tiles, starts from tile G' - which replaces tile G in Figure 9(iii) during the last application of Step 6. It then travels towards the east of tile G': If a tile has an active glue s_t on its west edge, this signal activates s_t on its east edge. In addition, for all tiles it encounters, s_t deactivates all glues from the south edge, and thus detaches the mirror rectangle from the output rectangle. The signal s_t is common to all tiles, and is not shown in these tables, for readability reasons. Signal s_1 is activated by tile C; the path that it travels is shown in green in Figure 9(iv). Once it reaches tile B, signal s_1 is converted to signal s_5. Tile E changes signal s_5 to s_6 (shown in black, respectively orange in Figure 12(i)), and s_6 starts the column detachment process. Signals s_7, s_8 (red), and s_9 (green), s_{10} (blue) in Figure 12(iii) and Figure 12(vii) are all used in column detachment, as follows. Signals s_7 and s_8 are used to travel the tiles above L. Signal s_9 deactivates the west glues of a column tile, while signal s_{10} deactivates the west glues of its neighbouring east tile, which belongs to the right part of the rectangle. Similarly, signal s_2 is the signal activated by the tile that mirrors C; the path it travels is shown in blue in Figure 9(iv), and it will lead to the column detachment in the mirror rectangle.

Tile Name	List of Signals				Transitions
	North	West	South	East	
L_5	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-,s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_6	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-,s_8^-, s_9^-, s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_7	s_8^-, s_9^-, s_{10}^-	s_{10}^-	s_7^-,s_8^-, s_9^-,s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_8	s_8^-, s_9^-	s_{10}^-	s_7^-,s_5^-, s_8^-, s_9^-,s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_9, N) \to \{(t, W), (x, W)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
L_9	s_8^-, s_9^-,s_{10}^-	s_{10}^-	s_7^-,s_8^-, s_9^-,s_{10}^-	s_{10}^-	$(s_7, S) \to \{(s_8, N)\}$, $(s_8, S) \to \{(s_9, S), (s_{10}, E)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$, $(s_{10}, N) \to \{(t, W), (x, W)\}$
M		s_{10}^-	s_{10}^-		$(s_{10}, W) \to \{(s_{10}, S)\}$
M_1		s_{10}^-	s_{10}^-		$(s_{10}, W) \to \{(s_{10}, S)\}$
M_2		s_{10}^-	s_4^-, s_{10}^-		$(s_4, S) \to \{(o, W), (x_2, W), (m_4, S), (x_1, E), (o, E)\}$, $(s_{10}, W) \to \{(s_{10}, S)\}$
M_3		s_{10}^-	s_4^+, s_{10}^-		$(s_{10}, W) \to \{(s_{10}, S)\}$

Table 2 Together with Table 1, this table list the signals and transitions for all the tiles types, except the dark grey border areas. See caption of Table 1 for details.

Note that, since all self-assemblies happen in parallel, it would in principle be possible for various rectangle components to assemble at the same time, bringing the potential of undesired attachments. During the entire construction of the thin rectangle, the existence of the mirror rectangle prevents the attachment of incorrect left and right parts of rectangles (with different number of rows, or erroneous number of columns), by posing a geometrical constraint.

More precisely, in Step 5 (reattachment) the left and the right parts of the corresponding rectangle(s) must reattach,, and this is where the possibility of an incorrect attachment occurs. Since the reattachment of the two parts happens through tile M_2 and its mirror tile in Θ' (marked in red in Figure 15), and since these two tiles only appear in the top row of the left part (the bottom row of the right part), the vertical distance between these two tiles in a potential attachment guarantees that the two parts of the rectangle(s) have the same number of rows. The horizontal distance between these two tiles in a potential attachment guarantees that the two parts of the rectangle(s) have the same number of columns in the interlocking portion. Together, these two conditions guarantee that the left and right parts of the rectangle(s) belong to the same step.

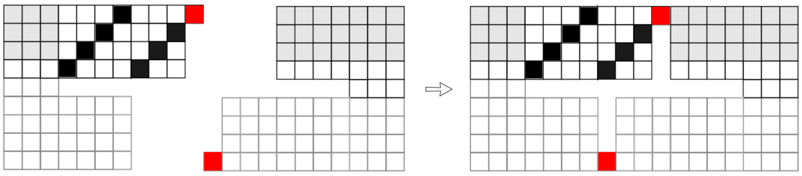

Fig. 15 The mirror rectangle guarantees correct reattachments in Step 5, through the geometrical constraints which it imposes on the attachment. The tiles marked in red are the only tiles through which the left and right parts of the rectangle(s) can reattach in Step 5.

4.3 DTAM tile complexity

The tile sets Θ and Θ' that are used to build the rectangles and mirror rectangles have a constant number of tiles. In order to construct an $N \times N!$ rectangle, we start from a small rectangle of size $6 \times (6! + 2\lceil \log(N-5) + 2 \rceil)$. Assuming that the assembly of this initial rectangle needs $O(\log(N))$ tile types, the entire construction will need a constant number of tiles types for Steps 2-4, and $O(\log(N))$ tile types to construct the dark grey borders, totalling $O(\log(N))$ tiles types.

[1] compares the tile complexities of various constructions of "thin rectangle". The construction of an $N \times N!$ rectangle using the most economic self-assembly models (multi-temperature self-assembly model, q-tile model, or unique shape model [1]) have a tile complexity of $O(\log N!/\log\log N!)$, that is, use approximately $O(N)$ tile types. In comparison, our DTAM self-assembly system utilizes only $O(\log N)$ tile types.

5 Conclusions

This paper introduces a simplified version of the Signal Tile Assembly Model (STAM), called DTAM (Detachable Tile Assembly Model) which is based on 2-HAM and uses a restricted version of signals, namely signals that can only deactivate glues (as opposed to the glue-activating and glue-deactivating signals of STAM). We show that a doubly-simplified version of STAM, namely SDTAM, that uses glue-deactivating signals only and, in addition, uses only one-tile-at-a-time attachments, is still capable of universal Turing computation at temperature one. Moreover, our Turing-simulating SDTAM uses at most one signal per tile, and signals travel through only one tile before deactivating a glue. All these simplifications, that are achieved without sacrificing any computational power, could have potential implications on the practical implementations of signal-based tile assembly systems. We also present a DTAM that assembles a thin $N \times N!$ rectangle, and has a lower tile-complexity than existing constructions. This illustrates the potential benefits of the DTAM model for tile-complexity reduction.

References

1. G. Aggarwal, Q. Cheng, M. Goldwasser, M.-Y. Kao, P. M. de Espanés, and R. Schweller. Complexities for generalized models of self-assembly. *SIAM J. Comput.*, 34(6):1493–1515, 2005.

2. B. Behsaz, J. Manuch, and L. Stacho. Turing universality of step-wise and stage assembly at temperature 1. In *DNA18*, volume 7433 of *LNCS*, pages 1–11, 2012.
3. H.-L. Chen, R. Schulman, A. Goel, and E. Winfree. Reducing facet nucleation during algorithmic self-assembly. *Nano Letters*, 7(9):2913–2919, 2007. PMID: 17718529.
4. M. Cook, Y. Fu, and R. Schweller. Temperature 1 self-assembly: Deterministic assembly in 3D and probabilistic assembly in 2D. In *SODA*, pages 570–589. SIAM, 2011.
5. E. Demaine, M. Demaine, S. Fekete, M. Ishaque, E. Rafalin, R. Schweller, and D. Souvaine. Staged self-assembly: nanomanufacture of arbitrary shapes with o(1) glues. *Natural Computing*, 7(3):347–370, 2008.
6. D. Doty, L. Kari, and B. Masson. Negative interactions in irreversible self-assembly. *Algorithmica*, 66(1):153–172, 2013.
7. T. Fochtman, J. Hendricks, J. Padilla, M. Patitz, and T. Rogers. Signal transmission across tile assemblies: 3D static tiles simulate active self-assembly by 2D signal-passing tiles. *Natural Computing*, 14(2):251–264, 2015.
8. A. Keenan, R. Schweller, and X. Zhong. Exponential replication of patterns in the signal tile assembly model. In *DNA19*, volume 8141 of *LNCS*, pages 118–132, 2013.
9. J. Padilla, W. Liu, and N. Seeman. Hierarchical self assembly of patterns from the Robinson tilings: DNA tile design in an enhanced Tile Assembly Model. *Natural Computing*, 11:323–338, 2012.
10. J. Padilla, M. Patitz, R. Pena, R. Schweller, N. Seeman, R. Sheline, S. Summers, and X. Zhong. Asynchronous signal passing for tile self-assembly: Fuel efficient computation and efficient assembly of shapes. In *UCNC 2013*, volume 7956 of *LNCS*, pages 174–185, 2013.
11. M. Patitz. An introduction to tile-based self-assembly and a survey of recent results. *Natural Computing*, 13(2):195–224, 2014.
12. J. Reif, S. Sahu, and P. Yin. Complexity of graph self-assembly in accretive systems and self-destructible systems. *Theor. Comput. Sci.*, 412(17):1592–1605, 2011.
13. P. Rothemund and E. Winfree. The program-size complexity of self-assembled squares (extended abstract). In *STOC*, pages 459–468, 2000.
14. P. Rothemund and E. Winfree. The program-size complexity of self-assembled squares (extended abstract). In *Proceedings of the Thirty-Second Annual ACM Symposium on Theory of Computing, May 21-23, 2000, Portland, OR, USA*, pages 459–468, 2000.
15. N. C. Seeman. Nucleic acid junctions and lattices. *Journal of Theoretical Biology*, 99(2):237 – 247, 1982.
16. E. Winfree. *Algorithmic self-assembly of DNA*. PhD thesis, 1998.
17. E. Winfree, F. Liu, L. Wenzler, and N. Seeman. Design and self-assembly of two-dimensional DNA crystals. *Nature*, 394:539–544, 1998.
18. E. Winfree, X. Yang, and N. Seeman. Universal computation via self-assembly of DNA: some theory and experiments. In *DNA Based Computers, Proceedings of a DIMACS Workshop, Princeton, New Jersey, USA, June 10-12, 1996*, pages 191–214, 1996.