Iterated Hairpin Completions of Non-crossing Words [*]

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Abstract. Iterated hairpin completion is an operation on formal languages that is inspired by the hairpin formation in DNA biochemistry. Iterated hairpin completion of a word (or more precisely a singleton language) is always a context-sensitive language and for some words it is known to be non-context-free. However, it is unknown whether regularity of iterated hairpin completion of a given word is decidable. Also the question whether iterated hairpin completion of a word can be context-free but not regular was asked in literature. In this paper we investigate iterated hairpin completions of non-crossing words and, within this setting, we are able to answer both questions. For non-crossing words we prove that the regularity of iterated hairpin completions is decidable and that if iterated hairpin completion of a non-crossing word is not regular, then it is not context-free either.

1 Introduction

On an abstract level, a DNA single strand can be viewed as a word over the four-letter alphabet $\{A, C, G, T\}$ where the letters represent the nucleobases adenine, cytosine, guanine, and thymine, respectively. The *Watson-Crick complement* of A is T and the complement of C is G. Two complementary single strands of opposite orientation can bond to each other and form a DNA double strand. Throughout the paper, we use the bar-notation for complementary strands of opposite orientation.

In the same manner, a single strand can bond to itself if two of its substrands are complementary and do not overlap with each other. Such an intramolecular

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base pairing is called a *hairpin*. We are especially interested in hairpins of single strands of the form $\sigma = \gamma\alpha\beta\overline{\alpha}$. Here, the substrand $\overline{\alpha}$ can bond to the substrand α. Then, by extension, a new single strand can be synthesized which we call a *hairpin completion* of σ, see Figure 1. In this situation we call the substrands that initiate the hairpin completion, α and $\overline{\alpha}$, *primers*.
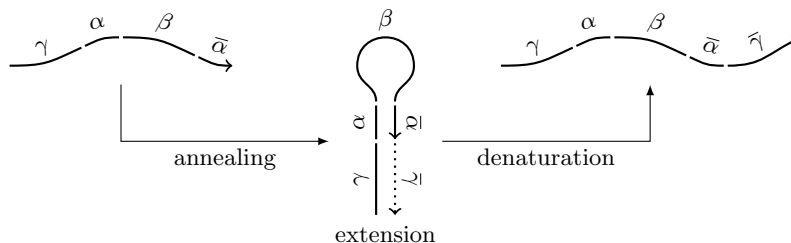

Fig. 1. Hairpin completion of a DNA single strand

In DNA computing hairpins and hairpin completions are often undesired by-products. Therefore, sets of strands have been analyzed and designed that are unlikely to form hairpins or lead to other *undesireable hybridization*, see [1, 2, 7, 8, 10, 18] and the references within.

However, there are DNA computational models that rely on hairpins, e. g., DNA RAM [9, 20, 21] and Whiplash PCR [5, 19, 22]. For the Whiplash PCR consider a single strand just like in Figure 1, but where the length of extension is controlled by *stopper sequences*. Repeating this operation, DNA can be used to solve combinatorial problems like the HAMILTONIAN PATH PROBLEM.

Inspired by hairpins in biocomputing, the hairpin completion of a formal language has been introduced by Cheptea, Martín-Vide, and Mitrana in [3]. In several papers hairpin completion and its iterated variant have been investigated, see [4, 12–17]. In this paper we consider iterated hairpin completions of singletons, that is, informally speaking, iterated hairpin completions of words. The class of iterated hairpin completions of singletons is denoted by HCS. It is known that every language in HCS is decidable in NL (non-deterministic, logarithmic space) as NL is closed under iterated hairpin completion [3]; hence, HCS is a proper subclass of the context-sensitive languages. It is also known that HCS contains regular as well as non-context-free languages [12]. In the latter paper, two open problems have been stated:

1. Is it decidable whether the iterated hairpin completion of a singleton is regular?
2. Does a singleton exist whose iterated hairpin completion is context-free but not regular?

We solve both questions for non-crossing words (or rather, singletons containing a non-crossing word). A word w is said to be non-crossing if, for a given

primer α, the right-most occurrence of the factor α in w precedes the left-most occurrence of the factor $\overline{\alpha}$ in w, see Section 3. We provide a necessary and sufficient condition for regularity of iterated hairpin completion of a given non-crossing word (Theorem 2 and Corollary 4) and, since this condition is decidable, we answer the first question positively (Corollary 5). Furthermore, we show that iterated hairpin completion of a non-crossing word is either regular or it is not context-free (Corollary 6). Thus, we give a negative answer to the second question.

This paper is the continuation of the studies in [11]. Due to the page limitation some proofs have been omitted. The missing proofs can be found in the appendix.

2 Preliminaries

We assume the reader to be familiar with the fundamental concepts of language theory, see [6].

Let Σ be an alphabet, Σ^* be the set of all words over Σ, and for an integer $k \geq 0$, Σ^k be the set of all words of length k over Σ. The word of length 0 is called the empty word, denoted by ε, and we let $\Sigma^+ = \Sigma^* \setminus \{\varepsilon\}$. A subset of Σ^* is called a language over Σ. For a word $w \in \Sigma^*$, we employ the notation w when we mean the word as well as the singleton language $\{w\}$ unless confusion arises.

We equip Σ with a function $^- : \Sigma \to \Sigma$ satisfying $\forall a \in \Sigma, \overline{\overline{a}} = a$; such a function is called an *involution*. This involution $^-$ is naturally extended to words as: for $a_1, \ldots, a_n \in \Sigma$, $\overline{a_1 a_2 \cdots a_n} = \overline{a_n} \cdots \overline{a_2} \overline{a_1}$. For a word $w \in \Sigma^*$, we call $\overline{w}$ the *complement of w*, being inspired by this application. A word $w \in \Sigma^*$ is called a *pseudo-palindrome* if $w = \overline{w}$. For a language $L \subseteq \Sigma^*$, we let $\overline{L} = \{\overline{w} \mid w \in L\}$.

For words $u, w \in \Sigma^*$, if $w = xuy$ holds for some words $x, y \in \Sigma^*$, then u is called a *factor* of w; a factor that is distinct from w is said to be *proper*. If the equation holds with $x = \varepsilon$ ($y = \varepsilon$), then the factor u is especially called a *prefix* (resp. a *suffix*) of w. The prefix relation can be regarded as a partial order $\leq_p$ over Σ^* whereas the proper prefix relation can be regarded as a strict order $<_p$ over Σ^*; $u \leq_p w$ means that u is a prefix of w and $u <_p w$ means that u is a proper prefix of w. Analogously, by $w \geq_s u$ (or $w >_s u$) we mean that u is a suffix (resp. proper suffix) of w. Note that $u \leq_p w$ if and only if $\overline{w} \geq_s \overline{u}$. For a word $w \in \Sigma^*$ and a language $L \subseteq \Sigma^*$, a factor u of w is *minimal with respect to L* if $u \in L$ and none of the proper factors of u is in L.

2.1 Hairpin Completion

Let k be a constant that is assumed to be the length of a primer and let $\alpha \in \Sigma^k$ be a primer. If a given word $w \in \Sigma^*$ can be written as $\gamma \alpha \beta \overline{\alpha}$ for some $\gamma, \beta \in \Sigma^*$, then its *right hairpin completion* (with respect to α) results in the word $\gamma \alpha \beta \overline{\alpha} \overline{\gamma}$. By $w \to_{\mathcal{RH}_\alpha} z$, we mean that z can be obtained from w by right hairpin completion (with respect to α). The *left hairpin completion* is defined analogously as an operation to derive $\gamma \alpha \beta \overline{\alpha} \overline{\gamma}$ from $\alpha \beta \overline{\alpha} \overline{\gamma}$, and the relation $\to_{\mathcal{LH}_\alpha}$ is naturally

introduced. We write $w \to_{\mathcal{H}_\alpha} z$ if $w \to_{\mathcal{RH}_\alpha} z$ or $w \to_{\mathcal{LH}_\alpha} z$. By $\to^*_{\mathcal{LH}_\alpha}$, $\to^*_{\mathcal{RH}_\alpha}$, and $\to^*_{\mathcal{H}_\alpha}$ we denote the reflexive and transitive closure of $\to_{\mathcal{LH}_\alpha}$, $\to_{\mathcal{RH}_\alpha}$, and $\to_{\mathcal{H}_\alpha}$, respectively. Whenever α is clear from the context, we omit the subscript α and write $\to_{\mathcal{RH}}$, $\to_{\mathcal{LH}}$, or $\to_{\mathcal{H}}$, respectively.

For a language $L \subseteq \Sigma^*$, we define the set of words obtained by hairpin completion from L, and the set of words obtained by iterated hairpin completion from L, respectively, as follows:

$$\mathcal{H}_\alpha(L) = \left\{ z \mid \exists w \in L, w \to_{\mathcal{H}_\alpha} z \right\}, \qquad \mathcal{H}_\alpha^*(L) = \left\{ z \mid \exists w \in L, w \to^*_{\mathcal{H}_\alpha} z \right\}.$$

In this paper the hairpin completion is always considered with respect to a fixed primer α. However, in other literature the hairpin completion is often considered with respect to the length k of primers instead of a specific primer α and defined as

$$\mathcal{H}_k(L) = \bigcup_{\alpha \in \Sigma^k} \mathcal{H}_\alpha(L), \qquad\qquad \mathcal{H}_k^*(L) = \bigcup_{i \geq 0} \mathcal{H}_k^i(L).$$

3   Non-crossing Words and Their Properties

In this section, we describe some structural properties of non-crossing words and their iterated hairpin completions and we introduce the notation of α-prefixes, α-suffixes, and α-indexes.

For a word α, we say that w is *non-α-crossing* if the rightmost occurrence of α precedes the leftmost occurrence of $\bar{\alpha}$ on w (yet these factors may overlap). If α is understood from the context, we simply say that w is *non-crossing*. Otherwise, the word is *α-crossing* or *crossing*. The definition of a word w being non-α-crossing becomes useful in our work only if $w \in \alpha\Sigma^*$ or $w \in \Sigma^*\bar{\alpha}$, and therefore, α and $\bar{\alpha}$ are primers; actually, we will assume both. The main purpose of this paper is to prove a necessary and sufficient condition for the regularity of the iterated hairpin completion $\mathcal{H}_\alpha^*(w)$, where $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ is non-α-crossing.

Note that if $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ and $\alpha = \bar{\alpha}$, then either $w = \alpha$ and $\mathcal{H}_\alpha^*(w) = \{w\}$ or w can be considered crossing. Thus, whenever we consider non-crossing words, we assume that $\alpha \neq \bar{\alpha}$.

Any word obtained from a non-crossing word by hairpin completion is non-crossing. Though being easily confirmed, this closure property forms the foundation of our discussions in this paper.

Proposition 1. *For a non-crossing word $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$, every word in $\mathcal{H}_\alpha^*(w)$ is non-crossing.*

Let us provide another characterization for a word $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ to be non-crossing. With Proposition 1, this characterization will bring a unique factorization of any word z in $\mathcal{H}_\alpha^*(w)$ as $z = xwy$ for some words x, y (Corollary 1).

Proposition 2. *A word $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ is non-crossing if and only if it contains exactly one factor x which is minimal with respect to $\alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$.*

Corollary 1. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ be non-crossing. The factor w occurs exactly once in every word from $\mathcal{H}_\alpha^*(w)$.*

3.1 α-Prefixes and $\bar{\alpha}$-Suffixes

Let u, v, w be words. We call u an α-*prefix* of w if $u\alpha \leq_p w$. This means, if $\bar{\alpha}$ is a suffix of w, then the suffix can bond to the factor α which directly follows the prefix u (unless they overlap with each other) and $w\bar{u}$ can be obtained from w by right hairpin completion. By $P_\alpha(w)$, we denote the set of all α-prefixes of w. Note that if $x, y \in P_\alpha(w)$ and $|x| \leq |y|$, then $x\alpha \leq_p y\alpha$ and $x \in P_\alpha(y\alpha)$. Symmetrically, we call $\bar{v}$ an $\bar{\alpha}$-*suffix* if $w \geq_s \bar{\alpha}\bar{v}$ and $S_{\bar{\alpha}}(w)$ is the set of all $\bar{\alpha}$-suffixes of w. If $\alpha \leq_p w$ and $|w| \geq |v| + 2k$, then $w \rightarrow_{\mathcal{LH}} vw$. Note that $S_{\bar{\alpha}}(w) = \overline{P_\alpha(\overline{w})}$. Therefore, all results we derive for α-prefixes also hold for the complements of $\bar{\alpha}$-suffixes.

When $m = |P_\alpha(w)|$ and $n = |S_{\bar{\alpha}}(w)|$ for a word w, then w is called (m, n)-α-*word* (or simply (m, n)-*word*). Throughout the paper, it will be convenient to let $P_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ and $S_{\bar{\alpha}}(w) = \{\overline{v_0}, \ldots, \overline{v_{n-1}}\}$ where the words are ordered such that $u_0 <_p \cdots <_p u_{m-1}$ and $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$. Note that $P_\alpha(u_i\alpha) = \{u_0, \ldots, u_i\}$ for $0 \leq i < m$ and $\overline{S_{\bar{\alpha}}(\overline{\alpha v_j})} = \{v_0, \ldots, v_j\}$ for $0 \leq j < n$.

Let us begin with a basic observation.

Lemma 1. *For a word $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$, the following statements hold:*

1. *For every $x \in P_\alpha(w) \cup \overline{S_{\bar{\alpha}}(w)}$, we have $\alpha \leq_p x\alpha$.*
2. *For every $x_1, \ldots, x_\ell \in P_\alpha(w) \cup \overline{S_{\bar{\alpha}}(w)}$, we have $\alpha \leq_p x_\ell \cdots x_1\alpha$.*

Consider $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$. Note that this means $u_0 = \overline{v_0} = \varepsilon$. It is easy to see that every word z which belongs to $\mathcal{H}_\alpha(w)$ has a factorization $z = w\bar{u}$ for some $u \in P_\alpha(w)$ or $z = vw$ for some $\bar{v} \in S_{\bar{\alpha}}(w)$. By the previous lemma we see that $z \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ and by induction $\mathcal{H}_\alpha^*(w) \subseteq \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$.

The next lemma tells if $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ is a non-crossing (m, n)-word with $n \geq 2$, then the suffix $\bar{\alpha}$ does not overlap with any of the factors α and, therefore, $w \rightarrow_{\mathcal{RH}} w\bar{u}$ for all $u \in P_\alpha(w)$.

Lemma 2. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ be a non-crossing (m, n)-word with $n \geq 2$ and let u_{m-1} be the longest α-prefix in $P_\alpha(w)$. Then $|u_{m-1}| + 2k \leq |w|$ holds.*

Since the analogous argument is valid for left hairpin completion, Lemma 2 leads us to one important corollary on non-crossing (m, n)-words for $m, n \geq 2$.

Corollary 2. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ be a non-crossing (m, n)-word with $m, n \geq 2$.*

$$\mathcal{H}_\alpha(w) = \{w\} \cup w\overline{P_\alpha(w)} \cup \overline{S_{\bar{\alpha}}(w)}w.$$

Next, we concern the case when there is a prefix $u \in P_\alpha(w)$ and a suffix $\bar{v} \in S_{\bar{\alpha}}(w)$ such that $v \in P_\alpha(u\alpha)^*$ and $u \in \overline{S_{\bar{\alpha}}(\overline{\alpha v})}^*$.

Lemma 3. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\bar{\alpha}$ be an (m, n)-word. For $u \in P_\alpha(w)$ and $\bar{v} \in S_{\bar{\alpha}}(w)$,*

1. *if $v \in P_\alpha(u\alpha)^*$, then $\overline{S_{\bar{\alpha}}(\overline{\alpha v})} \subseteq P_\alpha(u\alpha)^*$.*
2. *if $v \in P_\alpha(u\alpha)^*$ and $u \in \overline{S_{\bar{\alpha}}(\overline{\alpha v})}^*$, then $P_\alpha(u\alpha)^* = \overline{S_{\bar{\alpha}}(\overline{\alpha v})}^*$.*

Lemma 7 and Corollary 3 in in Section 4 will describe the consequences for the iterated hairpin completion of a non-crossing word w, if we find such a situation.

3.2 The α-Index

The α-*index* of a word x is the number of occurrences of the factor α in the word $x\alpha$ except for the suffix. Formally, we define a function $\mathrm{ind}_\alpha \colon \Sigma^* \to \mathbb{N}$ as

$$\mathrm{ind}_\alpha(x) = |\mathrm{P}_\alpha(x\alpha)| - 1.$$

Recall that $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$. Note that, by this ordering, for all $0 \leq i < m$ we have $\mathrm{ind}_\alpha(u_i) = i$ and if $x \in \mathrm{P}_\alpha(w)$, then $x = u_{\mathrm{ind}_\alpha(x)}$. Symmetrically, if $\bar{x} \in \mathrm{S}_{\bar\alpha}(w)$, then $x = v_{\mathrm{ind}_\alpha(x)}$.

Also note that for words x, y with $\mathrm{ind}_\alpha(x) > \mathrm{ind}_\alpha(y)$ the word x cannot be a factor of y as the positions of the factors α cannot match. Later we will consider the α-indices of words from $\alpha \Sigma^* \alpha^{-1}$ (Note that $x \in \alpha \Sigma^* \alpha^{-1}$ if and only if $\alpha \leq_p x\alpha$). If $x \in \alpha \Sigma^* \alpha^{-1}$ and $y \in \Sigma^*$, then $\mathrm{ind}_\alpha(yx) = \mathrm{ind}_\alpha(y) + \mathrm{ind}_\alpha(x)$. These observations lead to the following properties.

Lemma 4. *Let* $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ *such that* $u_0 <_p \cdots <_p u_{m-1}$, *let* $x \in \alpha \Sigma^* \alpha^{-1}$, *and let* $0 \leq j < m$. *If* x *is a suffix of* u_j, *then* $u_j = u_{j-\mathrm{ind}_\alpha(x)}x$. *In particular, if* $w \in \alpha \Sigma^*$ *and* $u_j \geq_s u_i$ *for* $0 \leq i \leq j < m$, *then* $u_j = u_{j-i}u_i$.

Lemma 5. *Let* $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ *such that* $u_0 <_p \cdots <_p u_{m-1}$, *let* $1 \leq i < m$, *and let* x *be a word with* $\mathrm{ind}_\alpha(x) \leq i$.

$$x \in \{u_1, \ldots, u_i\}^* \quad \Longleftrightarrow \quad x \in \{u_1, \ldots, u_{\mathrm{ind}_\alpha(x)}\}^*.$$

4 Iterated Hairpin Completion of Non-crossing Words

Now we are prepared to prove a necessary and sufficient condition for the regularity of $\mathcal{H}_\alpha^*(w)$, where $w \in \alpha \Sigma^* \cap \Sigma^* \bar\alpha$ is a non-crossing (m, n)-word with $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ and $\mathrm{S}_{\bar\alpha}(w) = \{\overline{v_0}, \ldots, \overline{v_{n-1}}\}$ which are ordered as in the previous section. (Keep in mind that $u_0 = \overline{v_0} = \varepsilon$.) By a result from [11] it is enough to consider the case where $m, n \geq 2$ and in this case, by Corollary 2,

$$\mathcal{H}_\alpha(w) = \{w\} \cup w\{\overline{u_1}, \ldots, \overline{u_{m-1}}\} \cup \{v_1, \ldots, v_{n-1}\}w.$$

Theorem 1 (See [11]). *If* $w \in \alpha \Sigma^* \cap \Sigma^* \bar\alpha$ *is a non-crossing (m, n)-word with* $m = 1$ *or* $n = 1$, *then* $\mathcal{H}_\alpha^*(w)$ *is regular.*

The next two lemmas lead to a first sufficient condition for the regularity of $\mathcal{H}_\alpha^*(w)$, see Corollary 3.

Lemma 6. *For non-crossing* $w \in \alpha \Sigma^* \cap \Sigma^* \bar\alpha$,

$$\mathcal{H}_\alpha^*(w) \subseteq \left(\mathrm{P}_\alpha(w) \cup \overline{\mathrm{S}_{\bar\alpha}(w)}\right)^* w \left(\overline{\mathrm{P}_\alpha(w)} \cup \mathrm{S}_{\bar\alpha}(w)\right)^*.$$

Lemma 7. *Let* $w \in \alpha \Sigma^* \cap \Sigma^* \bar\alpha$ *be non-crossing. If for some* $u \in \mathrm{P}_\alpha(w)$ *and* $\bar{v} \in \mathrm{S}_{\bar\alpha}(w)$ *we have* $v \in \mathrm{P}_\alpha(u\alpha)^*$ *and* $u \in \mathrm{S}_{\bar\alpha}(\bar\alpha\bar{v})^*$, *then*

$$\mathrm{P}_\alpha(u\alpha)^* w \overline{\mathrm{P}_\alpha(u\alpha)}^* = \overline{\mathrm{S}_{\bar\alpha}(\bar\alpha\bar{v})}^* w \mathrm{S}_{\bar\alpha}(\bar\alpha\bar{v})^* \subseteq \mathcal{H}_\alpha^*(w).$$

Suppose that the longest α-prefix u_{m-1} belongs to $\overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ and the longest $\overline{\alpha}$-suffix $\overline{v_{n-1}}$ belongs to $\overline{\mathrm{P}_\alpha(w)}^*$. By Lemma 3, we see that $\mathrm{P}_\alpha(w)^* = \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ and, by Lemmas 6 and 7, we infer $\mathcal{H}_\alpha^*(w) = \mathrm{P}_\alpha(w)^* w \overline{\mathrm{P}_\alpha(w)}^*$. (Note that $\mathrm{P}_\alpha(w) = \mathrm{P}_\alpha(u_{m-1}\alpha)$.)

Corollary 3. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\overline{\alpha}$ be non-crossing, let u_{m-1} be the longest α-prefix of w, and let v_{n-1} be the longest $\overline{\alpha}$-suffix of w. If $u_{m-1} \in \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ and $v_{n-1} \in \mathrm{P}_\alpha(w)^*$, then $\mathcal{H}_\alpha^*(w)$ is regular.*

Our next result, Theorem 2, shows that if we can state a necessary and sufficient condition for the special case where $u_1 = v_1$, then we can extend this condition to the general case. We need a preliminary lemma in order to prove Theorem 2.

Lemma 8. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\overline{\alpha}$ be a non-crossing (m,n)-word with $m, n \geq 2$, let $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$, and let $\mathrm{S}_{\overline{\alpha}}(w) = \{\overline{v_0}, \ldots, \overline{v_{v-1}}\}$ such that $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$.*

1. *If $u_1 = v_1$, then $\mathcal{H}_\alpha^*(w) \subseteq u_1\alpha\Sigma^* \cap \Sigma^*\overline{\alpha}\overline{u_1}$.*
2. *If $u_1 \neq v_1$, then $\mathcal{H}_\alpha^*(w\overline{u_i}) \cap \mathcal{H}_\alpha^*(v_j w) = \emptyset$ for all $1 \leq i < m$ and $1 \leq j < n$.*
3. *Let $1 \leq i < j < m$. If $u_j >_s u_i$, then $\mathcal{H}_\alpha^*(w\overline{u_j}) \subseteq \mathcal{H}_\alpha^*(w\overline{u_i})$; otherwise, $\mathcal{H}_\alpha^*(w\overline{u_i}) \cap \mathcal{H}_\alpha^*(w\overline{u_j}) = \emptyset$.*

Let us define the index sets

$$I = \left\{ i \mid 1 \leq i < m \wedge u_i \notin \{u_1, \ldots, u_{i-1}\}^* \right\},$$
$$J = \left\{ j \mid 1 \leq j < n \wedge v_j \notin \{v_1, \ldots, v_{j-1}\}^* \right\}.$$

Thus, for all $i \in I$, no proper suffix of u_i belongs to $\mathrm{P}_\alpha(w)$ and for all $j \in J$, no proper prefix of $\overline{v_j}$ belongs to $\mathrm{S}_{\overline{\alpha}}(w)$, see Lemma 4. By the previous lemma, if $v_1 \neq u_1$, then $\mathcal{H}_\alpha^*(w)$ is the disjoint union

$$\mathcal{H}_\alpha^*(w) = \{w\} \cup \bigcup_{i \in I} \mathcal{H}_\alpha^*(w\overline{u_i}) \cup \bigcup_{j \in J} \mathcal{H}_\alpha^*(v_j w).$$

Note that for every word $w\overline{u_i}$ with $i \in I$ or $v_j w$ with $j \in J$, the shortest α-prefix is complementary to the shortest $\overline{\alpha}$-suffix. This observation leads us to an important theorem that allows us to reduce the general case to the special case where $u_1 = v_1$.

Theorem 2. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\overline{\alpha}$ be a non-crossing (m,n)-word with $m, n \geq 2$, let $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$, let $\mathrm{S}_{\overline{\alpha}}(w) = \{\overline{v_0}, \ldots, \overline{v_{v-1}}\}$ such that $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$, and define I, J as above.*
For $u_1 \neq v_1$, $\mathcal{H}_\alpha^*(w)$ is regular if and only if $\mathcal{H}_\alpha^*(w\overline{u_i})$ is regular for all $i \in I$ and $\mathcal{H}_\alpha^*(v_j w)$ is regular for all $j \in J$.

Theorem 2 justifies the assumption $u_1 = v_1$ that we make from now on. The next two theorems prove a necessary and sufficient condition for the regularity of $\mathcal{H}_\alpha^*(w)$. We start by proving that the condition is sufficient.

Theorem 3. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\overline{\alpha}$ be a non-crossing (m,n)-word with $m, n \geq 2$, let $\mathrm{P}_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$, and let $\mathrm{S}_{\overline{\alpha}}(w) = \{\overline{v_0}, \ldots, \overline{v_{v-1}}\}$ such that $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$.*
$\mathcal{H}_\alpha^*(w)$ is regular if both of the following conditions hold:

1. for all $1 \leq s < m$, either $u_s \in \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ or $\overline{\mathrm{S}_{\overline{\alpha}}(w)}^* \subseteq \{u_1, \ldots, u_s\}^*$, and
2. for all $1 \leq t < n$, either $v_t \in \mathrm{P}_\alpha(w)^*$ or $\mathrm{P}_\alpha(w) \subseteq \{v_1, \ldots, v_t\}^* t$.

Proof. Assume that both the conditions 1 and 2 are satisfied. We may assume that there is $1 \leq s < m$ such that $u_s \notin \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ or there is $1 \leq t < n$ such that $v_t \notin \mathrm{P}_\alpha(w)^*$; otherwise Corollary 3 yields the regularity. In addition, we cannot assume the existence of both such s and t as $u_s \notin \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ implies $\overline{\mathrm{S}_{\overline{\alpha}}(w)}^* \subseteq \{u_1, \ldots, u_s\}^*$ due to the condition 1. The symmetry in the roles of conditions 1 and 2 enables us to assume that such s exists without loss of generality, and moreover, we can assume that for all $1 \leq i < s$, $u_i \in \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ and for all $s \leq i < m$, $u_i \notin \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ by Lemma 3.

Let $R' = \overline{\mathrm{S}_{\overline{\alpha}}(w)}^* w \{\overline{u_1}, \ldots, \overline{u_{s-1}}\}^*$ and for $s \leq i < m$, let

$$R_i = \{u_1, \ldots, u_i\}^* w \{\overline{u_1}, \ldots, \overline{u_i}\}^* \overline{u_i} \{\overline{u_1}, \ldots, \overline{u_i}\}^*.$$

Then we let $R = \bigcup_{s \leq i < m} R_i \cup R'$ and we claim $\mathcal{H}_\alpha^*(w) = R$.

Firstly, we prove that $R \subseteq \mathcal{H}_\alpha^*(w)$. Since $m, n \geq 2$, Corollary 2 can be used to see that $w \{\overline{u_1}, \ldots, \overline{u_i}\}^* \overline{u_i} \subseteq \mathcal{H}_\alpha^*(w)$ for $s \leq i < m$, and by Lemma 7,

$$R_i = \{u_1, \ldots, u_i\}^* w \{\overline{u_1}, \ldots, \overline{u_i}\}^* \overline{u_i} \{\overline{u_1}, \ldots, \overline{u_i}\}^* \subseteq \mathcal{H}_\alpha^*(w).$$

Consider $z \in R'$. We may factorize $z = x_i \cdots x_1 w \overline{y_1} \cdots \overline{y_j}$ where $x_1, \ldots, x_i \in \overline{\mathrm{S}_{\overline{\alpha}}(w)}$ and $y_1, \ldots, y_j \in \{u_1, \ldots, u_{s-1}\}$. Let $1 \leq t < n$ be minimal such that $v_t \notin \{u_1, \ldots, u_{s-1}\}^*$. As $v_t \in \{u_1, \ldots, u_s\}^*$, we see that u_s is a factor of v_t and $t \geq s$. By Lemma 5, $u_{s-1} \in \{v_1, \ldots, v_{t-1}\}^*$ and, by the minimality of t, $v_{t-1} \in \{u_1, \ldots, u_{s-1}\}^*$. If $\mathrm{ind}_\alpha(x_\ell) < t$ for all $1 \leq \ell \leq i$, then, due to Lemma 7,

$$z \in \{v_1, \ldots, v_{t-1}\}^* w \{\overline{v_1}, \ldots, \overline{v_{t-1}}\}^* \subseteq \mathcal{H}_\alpha^*(w).$$

Otherwise, let $1 \leq \ell \leq i$ be maximal such that $\mathrm{ind}_\alpha(x_\ell) \geq t$ and let $w' = x_\ell \cdots x_1 w$. Observe that $w \to_{\mathcal{LH}}^* w'$ and, again by Lemma 7,

$$z \in \{v_1, \ldots, v_{t-1}\}^* w' \{\overline{v_1}, \ldots, \overline{v_{t-1}}\}^* \subseteq \mathcal{H}_\alpha^*(w') \subseteq \mathcal{H}_\alpha^*(w).$$

Thus, $R \subseteq \mathcal{H}_\alpha^*(w)$.

Now we prove the opposite inclusion by induction on the length of a derivation to generate a word in $\mathcal{H}_\alpha^*(w)$ from w. Clearly, $w \in R$ (base case). As an induction hypothesis, we assume that any word which can be derived from w by $\ell - 1$ hairpin completions is in R, and consider $z \in \mathcal{H}_\alpha^*(w)$ whose shortest derivation from w by hairpin completions is of length ℓ. Let w' be the word that precedes z on this shortest derivation, that is, $w' \in \mathcal{H}_\alpha^{\ell-1}(w)$; hence, $w' \in R$ by the induction hypothesis. Therefore, w' must be either in R_i for some $s \leq i < m$ or in R'. Let us consider the first case first. If z is obtained from w' by right hairpin completion,

then the complement of the extended part is in $\{u_1, \ldots, u_i\}^* \{\varepsilon, u_1, \ldots, u_{m-1}\}$, and hence, $z \in R_j$ for some $i \le j < m$. Otherwise $(w' \to_{\mathcal{LH}} z)$, $z \in R_i$ because $\overline{S_{\overline{\alpha}}(w')} \subseteq \{u_1, \ldots, u_s\}^* \subseteq \{u_1, \ldots, u_i\}^*$. Next we consider the case when $w' \in R'$. Since $\{u_1, \ldots, u_{s-1}\}^* \subseteq \overline{S_{\overline{\alpha}}(w)}^*$ it is easy to see that if $w' \to_{\mathcal{LH}} z$, then $z \in R'$ as well. Otherwise $(w' \to_{\mathcal{RH}} z)$, $z \in w' \{\varepsilon, \overline{u_1}, \ldots, \overline{u_{m-1}}\} S_{\overline{\alpha}}(w)^*$ and, as $S_{\overline{\alpha}}(w)^* \subseteq \{\overline{u_1}, \ldots, \overline{u_s}\}^*$, if $z \notin R'$, this word is covered by some language R_i where $s \le i < m$.

Consequently, $\mathcal{H}_\alpha^*(w) = R$ is regular. $\qquad\square$

Theorem 4. *Let $w \in \alpha \Sigma^* \cap \Sigma^* \overline{\alpha}$ be a non-crossing (m,n)-word with $m, n \ge 2$, let $P_\alpha(w) = \{u_0, \ldots, u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$, let $S_{\overline{\alpha}}(w) = \{\overline{v_0}, \ldots, \overline{v_{v-1}}\}$ such that $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$, and let $u_1 = v_1$.*

1. *$\mathcal{H}_\alpha^*(w)$ is not regular if there are $1 \le s < m$ and $1 \le t < n$ such that $u_s \notin \{v_1, \ldots, v_{n-1}\}^*$ and $v_t \notin \{u_1, \ldots, u_s\}^*$.*
2. *$\mathcal{H}_\alpha^*(w)$ is not regular if there are $1 \le s < m$ and $1 \le t < n$ such that $u_s \notin \{v_1, \ldots, v_t\}^*$ and $v_t \notin \{u_1, \ldots, u_{m-1}\}^*$.*

Proof. Let s and t be the minimal indices that satisfy the conditions in statement 1. Note that $s, t \ge 2$ and $u_s, v_t \notin u_1^+$ as $u_1 = v_1$. We will first argue, why the assumption $s \le t$ is no restriction.

Let us consider the case where the conditions in statement 1 are satisfied, but the conditions in statement 2 are not satisfied. It is easy to see that $u_s \notin \{v_1, \ldots, v_t\}^*$ is satisfied anyway. By contradiction assume $s > t$. Due to Lemma 5,

$$v_t \notin \{u_1, \ldots, u_s\}^* \quad \Longleftrightarrow \quad v_t \notin \{u_1, \ldots, u_t\}^*$$
$$\Longleftrightarrow \quad v_t \notin \{u_1, \ldots, u_{m-1}\}^*.$$

This satisfies the conditions of statement 2 and yields the contradiction. We conclude $s \le t$.

Now, let us consider the case where the conditions of both statements are satisfied. Let s and t' be the minimal indices such that $u_s \notin \{v_1, \ldots, v_{n-1}\}^*$ and $v_{t'} \notin \{u_1, \ldots, u_{m-1}\}^*$. We may assume $s \le t'$, by symmetry. Note that $v_{t'-1} \in \{u_1, \ldots, u_{m-1}\}^*$, by the minimality of t'. If $v_{t'-1} \in \{u_1, \ldots, v_s\}^*$, then we see that s and $t = t'$ are the minimal indices that satisfy the conditions in statement 1 and $s \le t$. Otherwise, there is a factorization $v_{t'-1} = x u_i y$ where $x \in \{u_1, \ldots, u_s\}^*$, $s < i < m$, and $y \in \{u_1, \ldots, u_{m-1}\}^*$. Note that s and $t = \operatorname{ind}_\alpha(x) + s + 1$ (hence $v_t = x u_{s+1}$) are the minimal indices that satisfy the conditions of statement 1 and, obviously, $s \le t$.

Observe that the minimality of s yields $u_1, \ldots, u_{s-1} \in \{v_1, \ldots, v_{n-1}\}^*$. If $x \in \{u_1, \ldots, u_{s-1}, v_1, \ldots, v_{n-1}\}$ was a suffix of u_s, then $u_s = u_{s-\operatorname{ind}_\alpha(x)} x \in \{v_1, \ldots, v_{n-1}\}^*$, due to Lemma 4. Hence, none of these words is a suffix of u_s. Symmetrically, none of the words $u_1, \ldots, u_s, v_1, \ldots, v_{t-1}$ is a suffix of v_t. This observation will become crucial later.

We will now define a regular language R and show that the intersection $\mathcal{H}_\alpha^*(w) \cap R$ is not regular and, therefore, $\mathcal{H}_\alpha^*(w)$ is not regular either. We let

$$R = u_s u_1^{\ge n} v_t w \overline{u_1}^{\ge n} \overline{u_s}$$

and we claim

$$\mathcal{H}_\alpha^*(w) \cap R = \left\{ u_s u_1^\ell v_t w \overline{u_1}^\ell \overline{u_s} \mid \ell \geq n \right\} =: L,$$

which is obviously not regular. Note that for every $\ell \geq n$

$$w \to_{\mathcal{RH}}^* w\overline{u_1}^\ell \to_{\mathcal{RH}} w\overline{u_1}^\ell \overline{u_s} \to_{\mathcal{LH}} u_s u_1^\ell v_t w \overline{u_1}^\ell \overline{u_s}.$$

Hence, $\mathcal{H}_\alpha^*(w) \cap R \supseteq L$.

Let $z = u_s u_1^{\ell_1} v_t w \overline{u_1}^{\ell_2} \overline{u_s}$ for some $\ell_1, \ell_2 \geq n$, which is in R. We assume $z \in \mathcal{H}_\alpha^*(w)$ and prove that this assumption requires $\ell_1 = \ell_2$. Let z' be the *right-most* word in the derivation $w \to_{\mathcal{H}}^* z' \to_{\mathcal{H}}^* z$ such that $z' = xwy$ for some words x, y with $u_1^{\ell_1} v_t \geq_s x$ and $y \leq_p \overline{u_1}^{\ell_2}$; these conditions mean that x or y does not overlap with the prefix u_s or the suffix $\overline{u_s}$, respectively. By right-most we mean that either $z' \to_{\mathcal{LH}} x'z' \to_{\mathcal{H}}^* z$ where $x'x >_s u_1^{\ell_1} v_t$ or $z' \to_{\mathcal{RH}} z'y' \to_{\mathcal{H}}^* z$ where $\overline{u_1}^{\ell_2} <_p yy'$; this means x' or y' overlaps with the prefix u_s or the suffix $\overline{u_s}$, respectively. Obviously, $y \in \overline{u_1}^*$. Note that if $x \neq \varepsilon$, then x cannot be a proper suffix of v_t; otherwise a word from $u_1 = v_1, \ldots, v_{t-1}$ would be a suffix of v_t which was excluded. Hence, $x = \varepsilon$ or $x \in u_1^* v_t$.

First consider the case $z' \to_{\mathcal{LH}} x'z' \to_{\mathcal{H}}^* z$ where $x'x >_s u_1^{\ell_1} v_t$. We show that this case cannot occur. Let $u' \neq \varepsilon$ be the suffix of u_s such that $u' u_1^{\ell_1} v_t = x'x$. As $x' \in u_1^* \{v_1, \ldots, v_{n-1}\}$ and $u'\alpha \leq_p x'\alpha$, some word from $u_1 = v_1, \ldots, v_{n-1}$ would be a suffix of u_s.

Now consider the case $z' \to_{\mathcal{RH}} z'y' \to_{\mathcal{H}}^* z$ where $\overline{u_1}^{\ell_2} <_p yy'$. Again, let $u' \neq \varepsilon$ be the suffix of u_s such that $\overline{u_1}^{\ell_2} \overline{u'} = yy'$. As $u'\alpha$ is a prefix of $xu_{m-1}\alpha$ and none of the words $v_1, \ldots, v_t, u_1, \ldots, u_{s-1}$ is a suffix of u_s, we see that $u' = u_s$ and $x = \varepsilon$.

Thus, in order to generate z from w by iterated hairpin completion, the derivation process must be of the form

$$w \to_{\mathcal{RH}}^* w\overline{u_1}^{\ell_2} \overline{u_s} \to_{\mathcal{LH}}^* u_s u_1^{\ell_1} v_t w \overline{u_1}^{\ell_2} \overline{u_s} = z.$$

Let x be a (newly chosen) word such that $w\overline{u_1}^{\ell_2}\overline{u_s} \to_{\mathcal{LH}} xw\overline{u_1}^{\ell_2}\overline{u_s}$ is the first left hairpin completion in the derivation above. Therefore, $x\alpha$ is a prefix of $u_s u_1^{\ell_2} v_{n-1}\alpha$ and x is a suffix of $u_s u_1^{\ell_1} v_t$. In particular, every suffix y of x with $\text{ind}_\alpha(y) \leq t$ is a suffix of v_t. Recall that $\text{ind}_\alpha(u_s) = s \leq t$. If $x\alpha$ was a prefix of $u_s\alpha$, then some word from $u_1, \ldots, u_s$ would be a suffix of v_t which is impossible. Verify that $x \in u_s u_1^+$ and $x = u_s u_1^{\ell_2} v_j$ with $1 \leq j < t$ would also impose a forbidden suffix for v_t. Thus, we see that $x = u_s u_1^{\ell_2} v_j$ with $t \leq j < n$. The case $j > t$ is not possible as it implies $v_t\alpha <_p v_j\alpha = u_1^{j-t} v_t\alpha$ and a word from $u_1 = v_1, \ldots, v_{t-1}$ would be a suffix of v_t. Therefore, $x = u_s u_1^{\ell_2} v_t$ and since x is a suffix of $u_s u_1^{\ell_1} v_t$ and u_1 is not a suffix of u_s we deduce $u_s u_1^{\ell_2} v_t = u_s u_1^{\ell_1} v_t$.

Consequentially, $z \in \mathcal{H}_\alpha^*(w)$ if and only if $\ell_1 = \ell_2$. This completes the proof of $\mathcal{H}_\alpha^*(w) \cap R = L$. $\qquad\square$

Combining Theorems 3 and 4, we conclude:

Corollary 4. *Let $w \in \alpha\Sigma^* \cap \Sigma^*\overline{\alpha}$ be a non-crossing (m,n)-word with $m,n \geq 2$, let $\mathrm{P}_\alpha(w) = \{u_0,\dots,u_{m-1}\}$ such that $u_0 <_p \cdots <_p u_{m-1}$, let $\mathrm{S}_{\overline{\alpha}}(w) = \{\overline{v_0},\dots,\overline{v_{v-1}}\}$ such that $\overline{v_{n-1}} >_s \cdots >_s \overline{v_0}$, and let $u_1 = v_1$.*
$\mathcal{H}_\alpha^*(w)$ is regular if and only if

1. for all $1 \leq s < m$ we have $u_s \in \overline{\mathrm{S}_{\overline{\alpha}}(w)}^*$ or $\overline{\mathrm{S}_{\overline{\alpha}}(w)} \subseteq \{u_1,\dots,u_s\}^*$ and
2. for all $1 \leq t < n$ we have $v_t \in \mathrm{P}_\alpha(w)^*$ or $\mathrm{P}_\alpha(w) \subseteq \{v_1,\dots,v_t\}^*$.

Thus, we provided a necessary and sufficient condition for the regularity of a non-crossing (m,n)-word. As one can easily observe, this condition is decidable.

Corollary 5. *For a given non-α-crossing word w, it is decidable whether or not its iterated hairpin completion, $\mathcal{H}_\alpha^*(w)$, is regular.*

Furthermore, we can derive from the proof of Theorem 4 that if the iterated hairpin completion of w is not regular, then the intersection of $\mathcal{H}_\alpha^*(w)$ with $R = u_s u_1^{\geq n} v_t w \overline{u_1}^{\geq n} \overline{u_s}$ is not a regular language (for suitable s,t and in case $u_1 = v_1$). More precisely, we obtained the context-free language

$$\mathcal{H}_\alpha^*(w) \cap R = \left\{ u_s u_1^\ell v_t w \overline{u_1}^\ell \overline{u_s} \mid \ell \geq n \right\}.$$

Consider we intersect $\mathcal{H}_\alpha^*(w)$ with $R' = (u_s u_1^{\geq n} v_t)^2 w \overline{u_1}^{\geq n} \overline{u_s}$. Using the same arguments as we did within the proof of Theorem 4, we can show that

$$\mathcal{H}_\alpha^*(w) \cap R' = \left\{ u_s u_1^\ell v_t u_s u_1^\ell v_t w \overline{u_1}^\ell \overline{u_s} \mid \ell \geq n \right\},$$

which is a non-context-free language. Using this idea we can proof that if $\mathcal{H}_\alpha^*(w)$ is not regular, it is not context-free either. The details of this proof are left for the interested reader.

Corollary 6. *Let w be a non-α-crossing word. If its iterated hairpin completion $\mathcal{H}_\alpha^*(w)$ is not regular, then $\mathcal{H}_\alpha^*(w)$ is not context-free.*

Final Remarks

We prove that regularity of iterated hairpin completion a given of non-crossing word is decidable. The general case, including that of crossing words, remains to be explored.

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