Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

Chapter 2

With Question/Answer Animations

Chapter Summary

- Sets
  - The Language of Sets
  - Set Operations
  - Set Identities
- Functions
  - Types of Functions
  - Operations on Functions
- Sequences and Summations
  - Types of Sequences
  - Summation Formulae
- Matrices
  - Matrix Arithmetic

Sets

Section 2.1

Section Summary

- Definition of sets
- Describing Sets
  - Roster Method
  - Set-Builder Notation
- Some Important Sets in Mathematics
- Empty Set and Universal Set
- Subsets and Set Equality
- Venn diagrams
- Cardinality of Sets
- Tuples, Cartesian Product

Introduction

- Sets are one of the basic building blocks for the types of objects considered in discrete mathematics.
  - Important for counting.
  - Programming languages have set operations.
- Set theory is an important branch of mathematics.
  - Many different systems of axioms have been used to develop set theory.
  - Here we are not concerned with a formal set of axioms for set theory. Instead, we will use what is called naïve set theory.

Sets

- A *set* is an unordered collection of "objects", e.g. intuitively described by some property or properties (in *naïve set theory*)
  - the students in this class
  - the chairs in this room

- The objects in a set are called the *elements*, or *members* of the set. A set is said to *contain* its elements.
- The notation $a \in A$ denotes that a is an element of set A.
- If a is not a member of A, write $a \notin A$

Describing a Set: Roster Method

- $S = \{a,b,c,d\}$
- Order is not important

  $$S = \{a,b,c,d\} = \{b,c,a,d\}$$

- Each distinct object is either a member or not; listing more than once does not change the set.

  $$S = \{a,b,c,d\} = \{a,b,c,b,c,d\}$$

- *Ellipses* (…) may be used to describe a set without listing all of the members when the pattern is clear.

  $$S = \{a,b,c,d,\ldots,z\}$$

Roster Method

- Set of all vowels in the English alphabet:

  $V = \{a,e,i,o,u\}$

- Set of all odd positive integers less than 10:

  $O = \{1,3,5,7,9\}$

- Set of all positive integers less than 100:

  $S = \{1,2,3,........,99\}$

- Set of all integers less than 0:

  $S = \{...., -3,-2,-1\}$

Some Important Sets

N = *natural numbers* = {0,1,2,3….}

Z = *integers* = {…,-3,-2,-1,0,1,2,3,…}

Z^+ = *positive integers* = {1,2,3,…..}

Q = set of rational numbers

R = set of *real numbers*

R^+ = set of *positive real numbers*

C = set of *complex numbers*.

Set-Builder Notation

- Specify the property or properties that all members must satisfy:

$S = \{x \mid x \text{ is a positive integer less than } 100\}$

$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$

$O = \{x \in Z^+ \mid x \text{ is odd and } x < 10\}$

positive rational numbers:

$Q^+ = \{x \in \mathbf{R} \mid x = p/q, \text{ for some positive integers } p,q\}$

- A predicate may be used: $S = \{x \mid P(x)\}$
  - *Example*: $S = \{x \mid \text{Prime}(x)\}$

Interval Notation

$$[a,b] = \{x \mid a \le x \le b\}$$
$$[a,b) = \{x \mid a \le x < b\}$$
$$(a,b] = \{x \mid a < x \le b\}$$
$$(a,b) = \{x \mid a < x < b\}$$

closed interval [a,b]

open interval (a,b)

Truth Sets of Quantifiers

- Given a predicate P and a domain D, we define the *truth set* of P to be the set of elements in D for which $P(x)$ is true. The truth set of $P(\mathrm{x})$ is denoted by

$$\{x \in D | P(x)\}$$

- **Example**: The truth set of $P(x)$ where the domain is the integers and $P(x)$ is "$|x| = 1$" is the set $\{-1,1\}$

Sets can be elements of sets

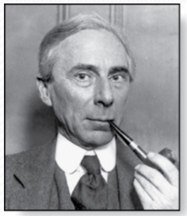
- Examples:

$$\{\{1,2,3\}, a, \{b,c\}\}$$

$$\{N,Z,Q,R\}$$

Russell's Paradox

- Let *S* be the set of all sets which are not members of themselves. A paradox results from trying to answer the question "Is *S* a member of itself?"



Bertrand Russell (1872-1970)
Cambridge, UK
Nobel Prize Winner

Related simple example:

- Henry is a barber who shaves all people who do not shave themselves. A paradox results from trying to answer the question "Does Henry shave himself?"

NOTE: To avoid this and other paradoxes, *sets* can be (formally) defined via appropriate axioms more carefully than just *an unordered collection of "objects"*

(where objects are intuitively described by any given property in *naïve set theory*)

U $\emptyset$

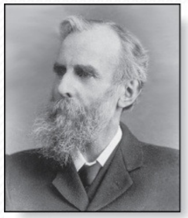
Universal Set and *Empty Set*

- The *universal set* is the set containing all the "objects" currently under consideration.
  - Often symbolized by *U*
  - Sometimes implicit
  - Sometimes explicitly stated.
  - Contents depend on the context.

- The *empty set* is the set with no elements.
  - Symbolized by ∅, but {} is also used.
  - NOTE: the empty set is different from a set containing the empty set.

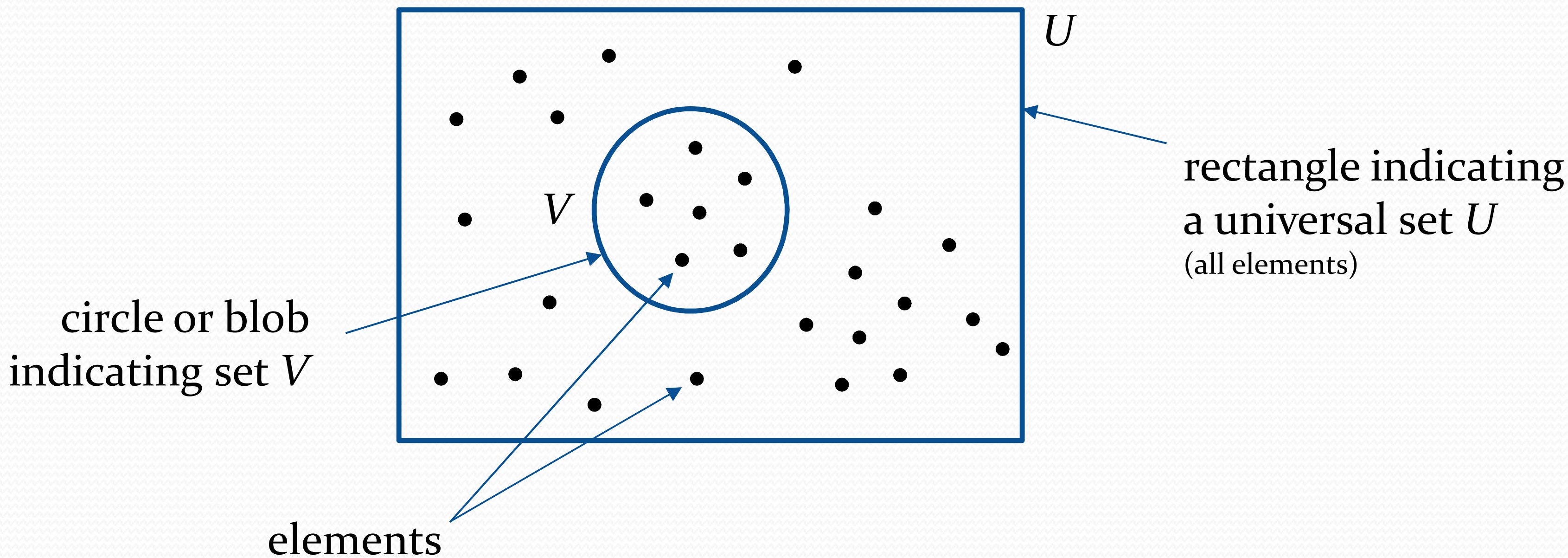
$$\emptyset \neq \{ \emptyset \}$$

Venn Diagram

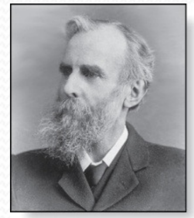


John Venn (1834-1923)
Cambridge, UK

- Sets and their elements can be represented via Venn diagrams



Venn Diagram



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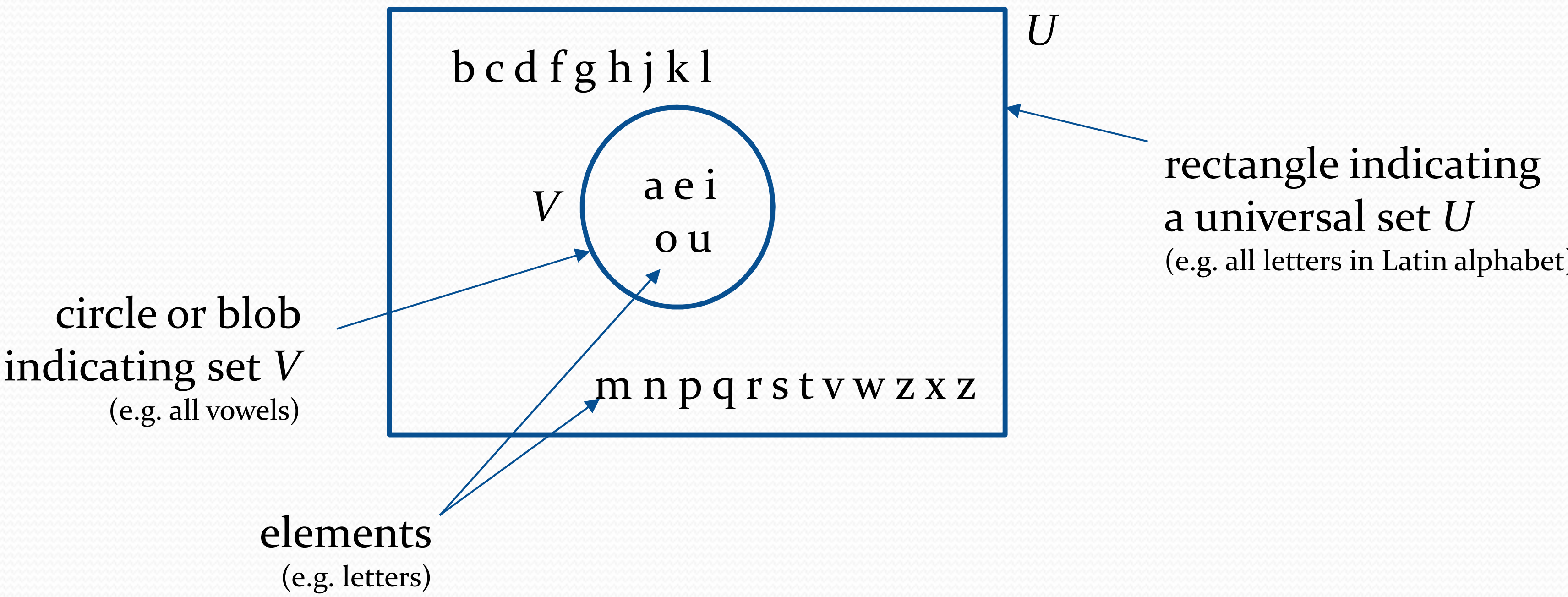
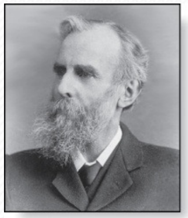


Venn Diagram



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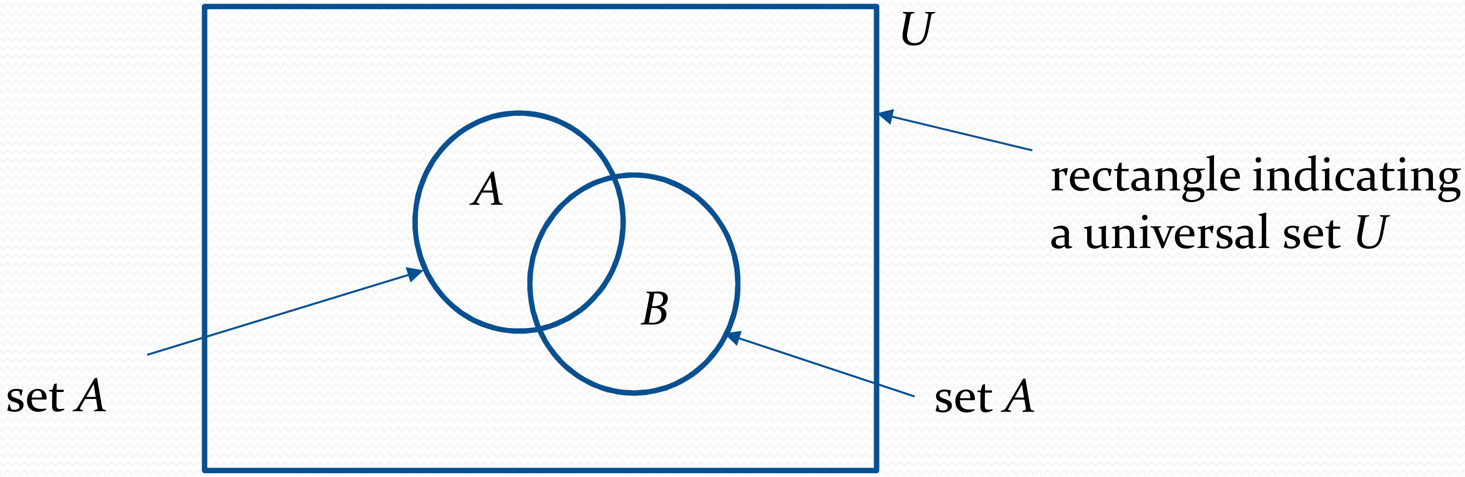
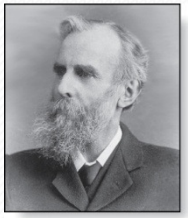
- Sets and their elements can be represented via Venn diagrams



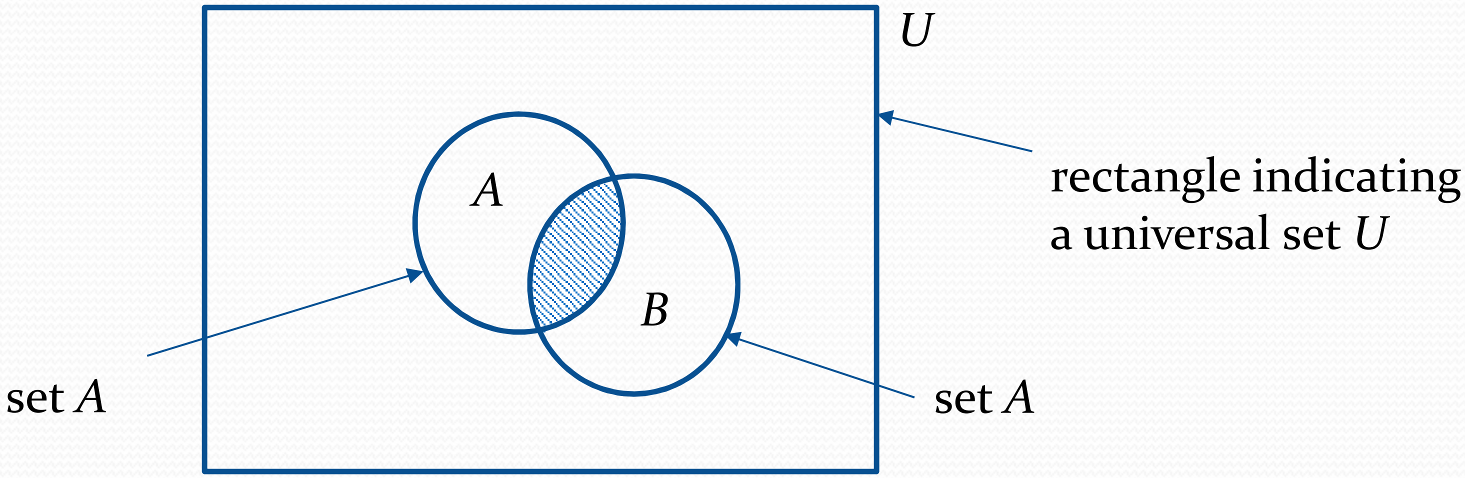
Venn diagrams are often drawn to abstractly illustrate relations between multiple sets.
Elements are implicit/omitted (shown as dots only when an explicit element is needed)

Venn Diagram



John Venn (1834-1923)
Cambridge, UK

- Sets and their elements can be represented via Venn diagrams



Example: shaded area illustrates a set of elements that are in both sets A and B (i.e. *intersection* of two sets, see later). E.g consider A={a,b,c,f,z} and B={c,d,e,f,x,y}.

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- If A and B are sets, then A and B are equal iff

$$\forall x(x \in A \leftrightarrow x \in B)$$

- We write $A = B$ if A and B are equal sets.

$$\{1,3,5\} = \{3, 5, 1\}$$

$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Subsets

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B
- $A \subseteq B$ holds if and only if $\forall x(x \in A \rightarrow x \in B)$ is true.
- NOTE:
  1. Because $a \in \emptyset$ is always false, $\emptyset \subseteq S$ for every set S.
  2. Because $a \in S \rightarrow a \in S$, $S \subseteq S$ for every set S.

Showing a Set is or is not a Subset of Another Set

- **Showing that A is a Subset of B**: To show that $A \subseteq B$, show that if x belongs to A, then x also belongs to B.
- **Showing that A is not a Subset of B**: To show that A is not a subset of B, $A \nsubseteq B$, find an element $x \in A$ with $x \notin B$. (Such an x is a counterexample to the claim that $x \in A$ implies $x \in B$.)

Examples:

1. The set of all computer science majors at your school is a subset of all students at your school.
2. The set of integers with squares less than 100 is not a subset of the set of nonnegative integers.

Another look at Equality of Sets

- Recall that two sets A and B are *equal* $(A = B)$ iff

$$\forall x (x \in A \leftrightarrow x \in B)$$

- That is, using logical equivalences we have that $A = B$ iff

$$\forall x [(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$$

- This is equivalent to

$$A \subseteq B \qquad \text{and} \qquad B \subseteq A$$

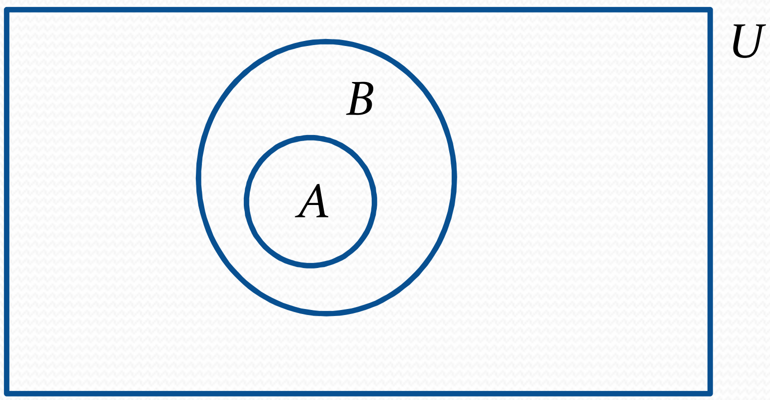
Proper Subsets

Definition: If $A \subseteq B$, but $A \neq B$, then we say A is a *proper subset* of B, denoted by $A \subset B$. If $A \subset B$, then

$$\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \in B \wedge x \notin A)$$

is true.

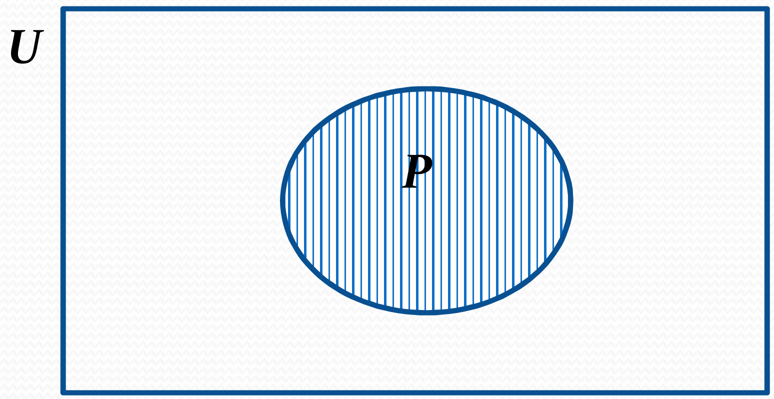
Venn Diagram
for a proper subset
$A \subset B$

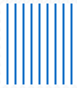


Example: A={c,f,z} and B={a,b,c,d,e,f,t,x,z}.

Venn Diagram and Truth Sets

Consider any predicate P(x) for elements x in U and its truth set P = { x | P(x) }.





- truth set P = { x | P(x) }

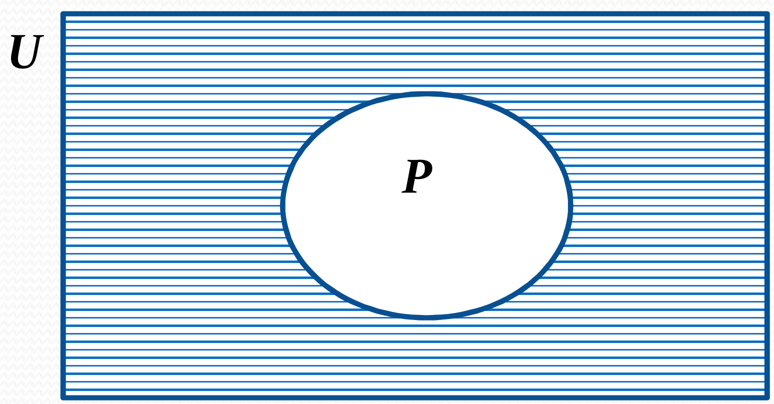
that is, all elements x where P(x) is true

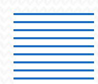
NOTE: $x \in P \equiv P(x)$

(negations)

Venn Diagram and Logical Connectives

Consider any predicate P(x) for elements x in U and its truth set P = { x | P(x) }.





- truth set $\{ x \mid \neg P(x) \}$

all elements x
where ¬ P(x) is true,
i.e. where P(x) is false

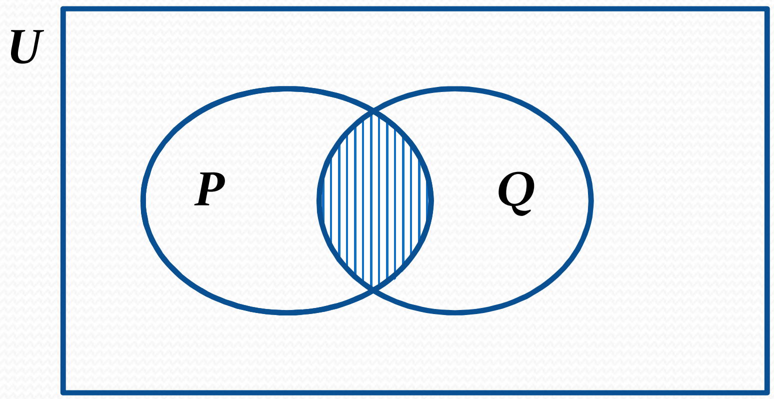
NOTE: $x \notin P \equiv \neg P(x)$

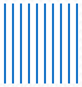
Same as *complement* of set P
(see section 2.2)

(conjunctions)

Venn Diagram and Logical Connectives

Consider arbitrary predicates P(x) and Q(x) defined for elements x in U and their corresponding truth sets P = { x | P(x) } and Q = { x | Q(x) }.



 - truth set $\{ x \mid P(x) \wedge Q(x) \}$

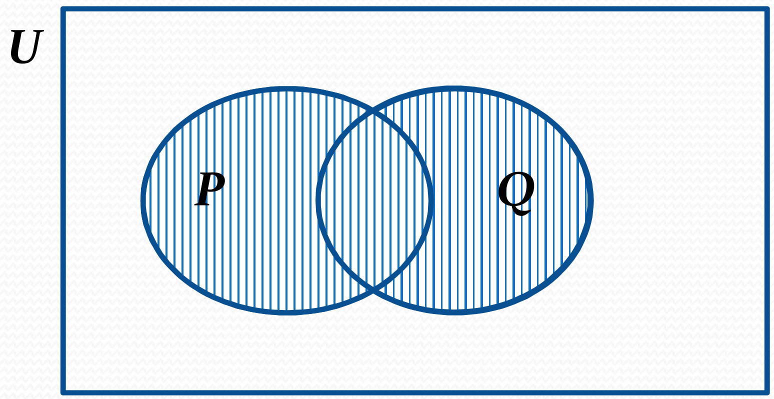
all elements x
where both P(x) and Q(x) is true

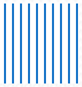
Same as *intersection* of sets P and Q
(see section 2.2)

(disjunctions)

Venn Diagram and Logical Connectives

Consider arbitrary predicates P(x) and Q(x) defined for elements x in U and their corresponding truth sets P = { x | P(x) } and Q = { x | Q(x) }.



 - truth set $\{ x \mid P(x) \vee Q(x) \}$

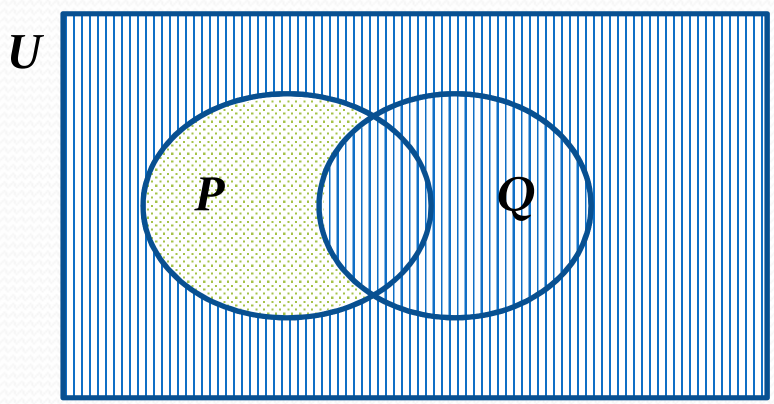
all elements x
where P(x) or Q(x) is true

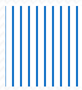
Same as *union* of sets P and Q
(see section 2.2)

(implications)

Venn Diagram and Logical Connectives

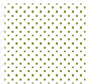
Consider arbitrary predicates P(x) and Q(x) defined for elements x in U and their corresponding truth sets $P = \{ x \mid P(x) \}$ and $Q = \{ x \mid Q(x) \}$.





- truth set $\{ x \mid P(x) \rightarrow Q(x) \}$

(all x where implication $P(x) \rightarrow Q(x)$ is true)



- set where implication $P(x) \rightarrow Q(x)$ is false:

$\{ x \mid \neg(\neg P(x) \vee Q(x)) \} = \{ x \mid P(x) \wedge \neg Q(x) \}$

$x \in P \wedge x \notin Q$

Remember: $p \rightarrow q \equiv \neg p \vee q$

Thus, $\{ x \mid P(x) \rightarrow Q(x) \} = \{ x \mid \neg P(x) \vee Q(x) \} = \{ x \mid x \notin P \vee x \in Q \}$

important special case

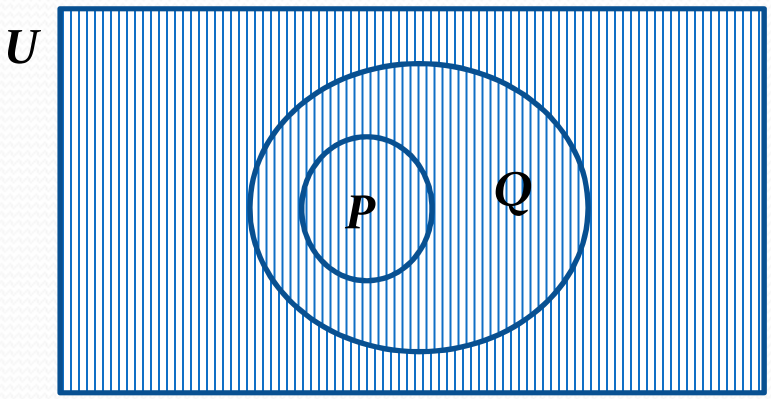
(implications)

Venn Diagram and Logical Connectives

Assume that it is known/proven that implication $P(x) \rightarrow Q(x)$ is true <u>for all x</u>.

That is, assume $\{ x \mid P(x) \rightarrow Q(x) \} \equiv U$ or that $\forall x \, (P(x) \rightarrow Q(x))$ is true.

Note, $\boxed{\forall x \, (P(x) \rightarrow Q(x))} \equiv \forall x \, (x \in P \rightarrow x \in Q) \equiv \boxed{P \subseteq Q}$ (directly from the definition of subsets)



NOTE: Venn diagram for $P \subseteq Q$
often shows P as a proper subset $P \subset Q$
(making "default" assumption $P \neq Q$)

- truth set $\{ x \mid P(x) \rightarrow Q(x) \}$
  (all x where implication $P(x) \rightarrow Q(x)$ is true)

- set where implication $P(x) \rightarrow Q(x)$ is false:
  is empty in this case
  $\{ x \mid x \in P \wedge x \notin Q \} = \varnothing$

formal proof ?

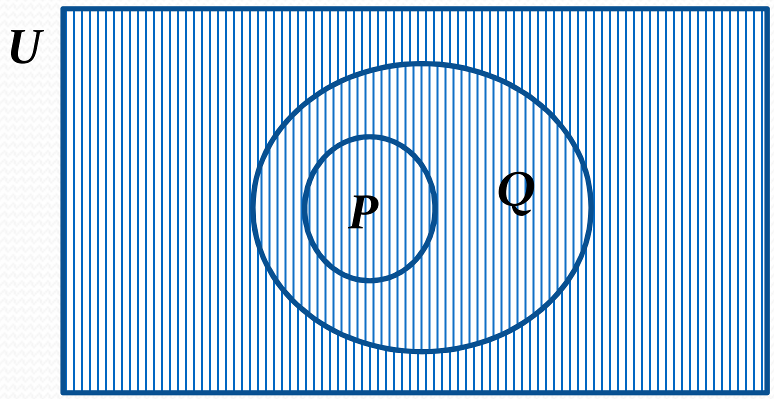
$$\forall x \, (P(x) \rightarrow Q(x)) \equiv \forall x \, (\neg P(x) \vee Q(x)) \equiv \neg \exists x \, (P(x) \wedge \neg Q(x)) \equiv \neg \exists x \, (x \in P \wedge x \notin Q)$$

important special case

(implications)

Venn Diagram and Logical Connectives

Note, $\forall x\ (P(x) \rightarrow Q(x)) \equiv \forall x\ (x \in P \rightarrow x \in Q) \equiv P \subseteq Q$ (directly from the definition of subsets)



- truth set $\{ x \mid P(x) \rightarrow Q(x) \}$

assuming $\forall x\ (P(x) \rightarrow Q(x))$ is true

This gives intuitive interpretation for logical "implications":

Proving theorems of the form $\forall x\ (P(x) \rightarrow Q(x))$ is equivalent to proving the subset relationship for the truth sets $P \subseteq Q$

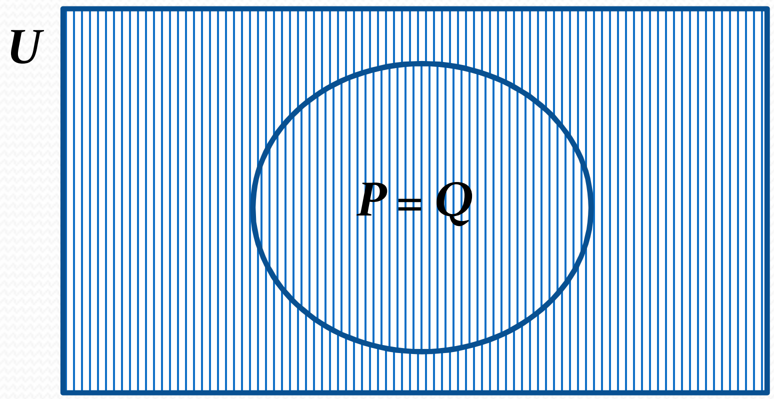
important special case

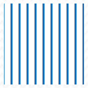
(biconditional)

Venn Diagram and Logical Connectives

Similarly one can show that $\forall x \ P(x) \leftrightarrow Q(x)$ $\equiv$ $P = Q$

that is, $\forall x \ (P(x) \rightarrow Q(x) \wedge Q(x) \rightarrow P(x))$ $\equiv$ $P \subseteq Q \ \wedge \ Q \subseteq P$




- truth set $\{ x \mid P(x) \leftrightarrow Q(x) \}$

assuming $\forall x \ (P(x) \leftrightarrow Q(x))$ is true

This gives intuitive interpretation for "biconditional":

Proving theorems of the form $\forall x \ (P(x) \leftrightarrow Q(x))$ is equivalent to proving the subset relationship for the truth sets $P = Q$

Set Cardinality

Definition: If there are exactly n (distinct) elements in S where n is a nonnegative integer, we say that S is *finite*. Otherwise it is *infinite*.

Definition: The *cardinality* of a finite set A, denoted by $|A|$, is the number of (distinct) elements of A.

Examples:

1. $|\varnothing| = 0$
2. Let S be the letters of the English alphabet. Then $|S| = 26$
3. $|\{1,2,3\}| = 3$
4. $|\{\varnothing\}| = 1$
5. The set of integers is infinite.

Power Sets

Definition: The set of all subsets of a set A, denoted P(A), is called the *power set* of A.

Example: If A = {a,b} then

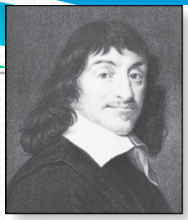
$$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$$

- If a set has n elements, then the cardinality of the power set is 2^n. (In Chapters 5 and 6, we will discuss different ways to show this.)

Tuples

- The *ordered n-tuple* $(a_1, a_2, \ldots\ldots, a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.
- Two n-tuples are equal if and only if their corresponding elements are equal.

- 2-tuples are called *ordered pairs*, e.g. (a_1, a_2)
- The ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.



René Descartes
(1596-1650)

Cartesian Product

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

$$A \times B = \{(a,b)|a \in A \wedge b \in B\}$$

Example:

$A = \{a,b\} \quad B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

- **Definition**: A subset R of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B. (Relations will be covered in depth in Chapter 9.)

Cartesian Product

Definition: The Cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for $i = 1, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times C$ where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$

Set Operations

Section 2.2

Section Summary

- Set Operations
  - Union
  - Intersection
  - Complementation
  - Difference
- More on Set Cardinality
- Set Identities
- Proving Identities
- Membership Tables

Boolean Algebra

- Propositional calculus and set theory are both instances of an algebraic system called a *Boolean Algebra*. This is discussed in CS2209.
- The operators in set theory are analogous to the corresponding operator in propositional calculus.
- As always there must be a universal set U. All sets are assumed to be subsets of U.

Union

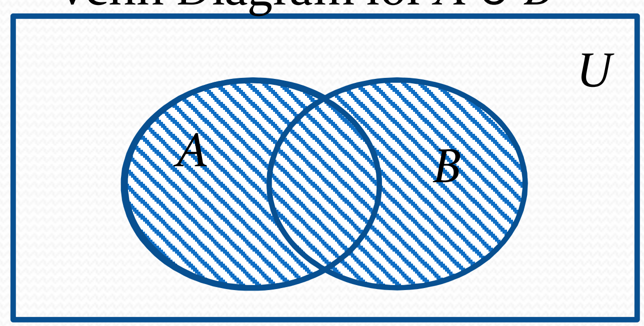
- **Definition**: Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set:

$$\{x | x \in A \vee x \in B\}$$

- **Example**: What is $\{1,2,3\} \cup \{3, 4, 5\}$?

Solution: $\{1,2,3,4,5\}$

Venn Diagram for $A \cup B$



Union is analogous to disjunction, see earlier slides.

Intersection

- **Definition**: The *intersection* of sets A and B, denoted by $A \cap B$, is

$$\{x | x \in A \wedge x \in B\}$$

- If the intersection is empty, then A and B are said to be *disjoint*.

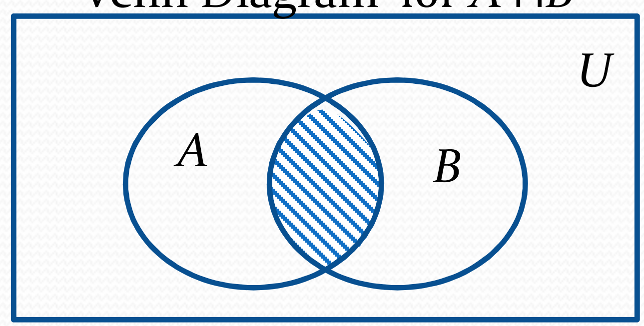
- **Example**: What is $\{1,2,3\} \cap \{3,4,5\}$?

  Solution: $\{3\}$

- **Example:** What is $\{1,2,3\} \cap \{4,5,6\}$?

  Solution: $\emptyset$

Venn Diagram for $A \cap B$



Intersection is analogous to conjunction, see earlier slides.

Complement

Definition: If A is a set, then the complement of the A (with respect to U), denoted by $\bar{A}$ is the set

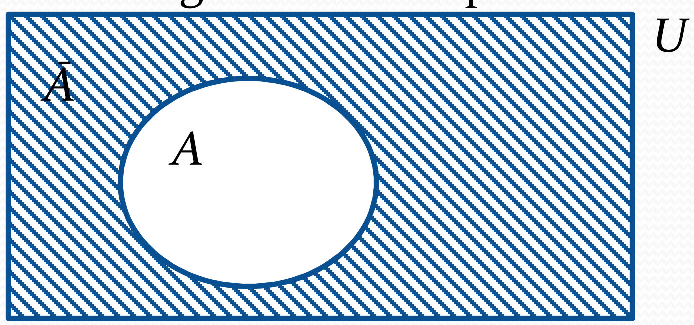
$$\bar{A} = \{x \in U \mid x \notin A\}$$

(The complement of A is sometimes denoted by A^c.)

Example: If U is the positive integers less than 100, what is the complement of $\{x \mid x > 70\}$

Solution: $\{x \mid x \leq 70\}$

Venn Diagram for Complement



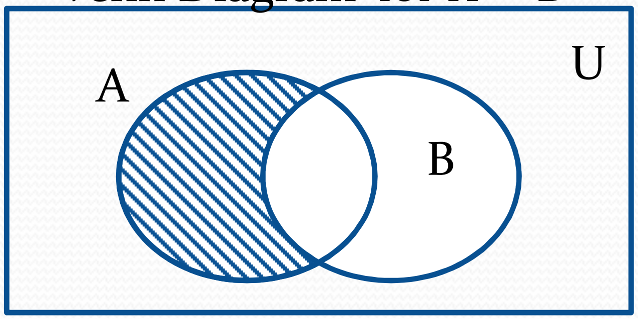
Complement is analogous to negation, see earlier.

Difference

- **Definition**: Let A and B be sets. The *difference* of A and B, denoted by $A - B$, is the set containing the elements of A that are not in B. The difference of A and B is also called the complement of B with respect to A.

$$A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$$

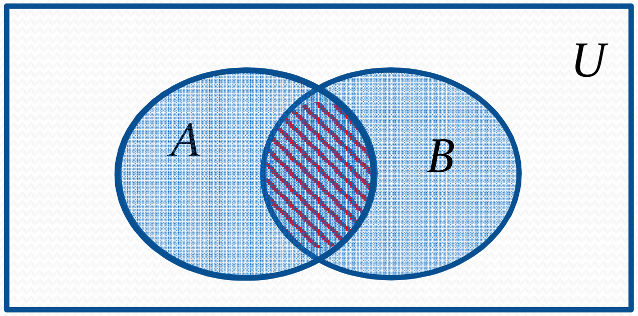
Venn Diagram for $A - B$



NOTE: $\bar{A} = U - A$

The Cardinality of the Union of Two Sets

- Inclusion-Exclusion
  $|A \cup B| = |A| + |B| - |A \cap B|$



Venn Diagram for A, B, $A \cap B$, $A \cup B$

- **Example**: Let A be the math majors in your class and B be the CS majors in your class. To count the number of students in your class who are either math majors or CS majors, add the number of math majors and the number of CS majors, and subtract the number of joint CS/math majors.
- We will return to this principle in Chapter 6 and Chapter 8 where we will derive a formula for the cardinality of the union of n sets, where n is a positive integer.

Review Questions

Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

1. $A \cup B$
2. $A \cap B$
3. $\bar{A}$
4. $\bar{B}$
5. $A - B$
6. $B - A$

Review Questions

Example: $U = \{0,1,2,3,4,5,6,7,8,9,10\}$ $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$

1. $A \cup B$

   Solution: $\{1,2,3,4,5,6,7,8\}$
2. $A \cap B$

   Solution: $\{4,5\}$
3. $\bar{A}$

   Solution: $\{0,6,7,8,9,10\}$
4. $\bar{B}$

   Solution: $\{0,1,2,3,9,10\}$
5. $A - B$

   Solution: $\{1,2,3\}$
6. $B - A$

   Solution: $\{6,7,8\}$

Set Identities

- Identity laws

$$A \cup \emptyset = A \qquad A \cap U = A$$

- Domination laws

$$A \cup U = U \qquad A \cap \emptyset = \emptyset$$

- Idempotent laws

$$A \cup A = A \qquad A \cap A = A$$

- Complementation law

$$\overline{(\overline{A})} = A$$

Continued on next slide →

Set Identities

- Commutative laws

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

- Associative laws

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Continued on next slide →

Set Identities

- De Morgan's laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

- Absorption laws

$$A \cup (A \cap B) = A \qquad A \cap (A \cup B) = A$$

- Complement laws

$$A \cup \overline{A} = U \qquad A \cap \overline{A} = \emptyset$$

Proving Set Identities

- Different ways to prove set identities:
  1. Prove that each set (i.e. each side of the identity) is a subset of the other.
  2. Use set builder notation and propositional logic.
  3. Membership Tables

(to be explained)

Proof of Second De Morgan Law

Example: Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Solution: We prove this identity by showing that:

1) $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ and

2) $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Continued on next slide →

Proof of Second De Morgan Law

These steps show that: $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

$x \in \overline{A \cap B}$	by assumption
$x \notin A \cap B$	defn. of complement
$\neg((x \in A) \wedge (x \in B))$	defn. of intersection
$\neg(x \in A) \vee \neg(x \in B)$	1st De Morgan Law for Prop Logic
$x \notin A \vee x \notin B$	defn. of negation
$x \in \overline{A} \vee x \in \overline{B}$	defn. of complement
$x \in \overline{A} \cup \overline{B}$	defn. of union

Continued on next slide →

Proof of Second De Morgan Law

These steps show that: $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

$x \in \overline{A} \cup \overline{B}$	by assumption
$(x \in \overline{A}) \vee (x \in \overline{B})$	defn. of union
$(x \notin A) \vee (x \notin B)$	defn. of complement
$\neg(x \in A) \vee \neg(x \in B)$	defn. of negation
$\neg((x \in A) \wedge (x \in B))$	by 1st De Morgan Law for Prop Logic
$\neg(x \in A \cap B)$	defn. of intersection
$x \in \overline{A \cap B}$	defn. of complement

◀

Set-Builder Notation: Second De Morgan Law

$$
\begin{array}{rcll}
\overline{A \cap B} & = & \{x | x \notin A \cap B\} & \text{by defn. of complement} \\
 & = & \{x | \neg(x \in (A \cap B))\} & \text{by defn. of does not belong symbol} \\
 & = & \{x | \neg(x \in A \land x \in B\} & \text{by defn. of intersection} \\
 & = & \{x | \neg(x \in A) \lor \neg(x \in B)\} & \text{by 1st De Morgan law} \\
 & & & \text{for Prop Logic} \\
 & = & \{x | x \notin A \lor x \notin B\} & \text{by defn. of not belong symbol} \\
 & = & \{x | x \in \overline{A} \lor x \in \overline{B}\} & \text{by defn. of complement} \\
 & = & \{x | x \in \overline{A} \cup \overline{B}\} & \text{by defn. of union} \\
 & = & \overline{A} \cup \overline{B} & \text{by meaning of notation}
\end{array}
$$

Membership Table

Example: Construct a membership table to show that the distributive law holds.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution:

A	**B**	**C**	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Generalized Unions and Intersections

- Let A_1, A_2 ,…, A_n be an indexed collection of sets.

  We define:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

These are well defined, since union and intersection are associative.

- *Example*: for i = 1,2,…, let A_i = {i, i + 1, i + 2, ….}. Then,

$$\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} \{i, i+1, i+2, \ldots\} = \{1, 2, 3, \ldots\}$$

$$\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n} \{i, i+1, i+2, \ldots\} = \{n, n+1, n+2, \ldots..\} = A_n$$