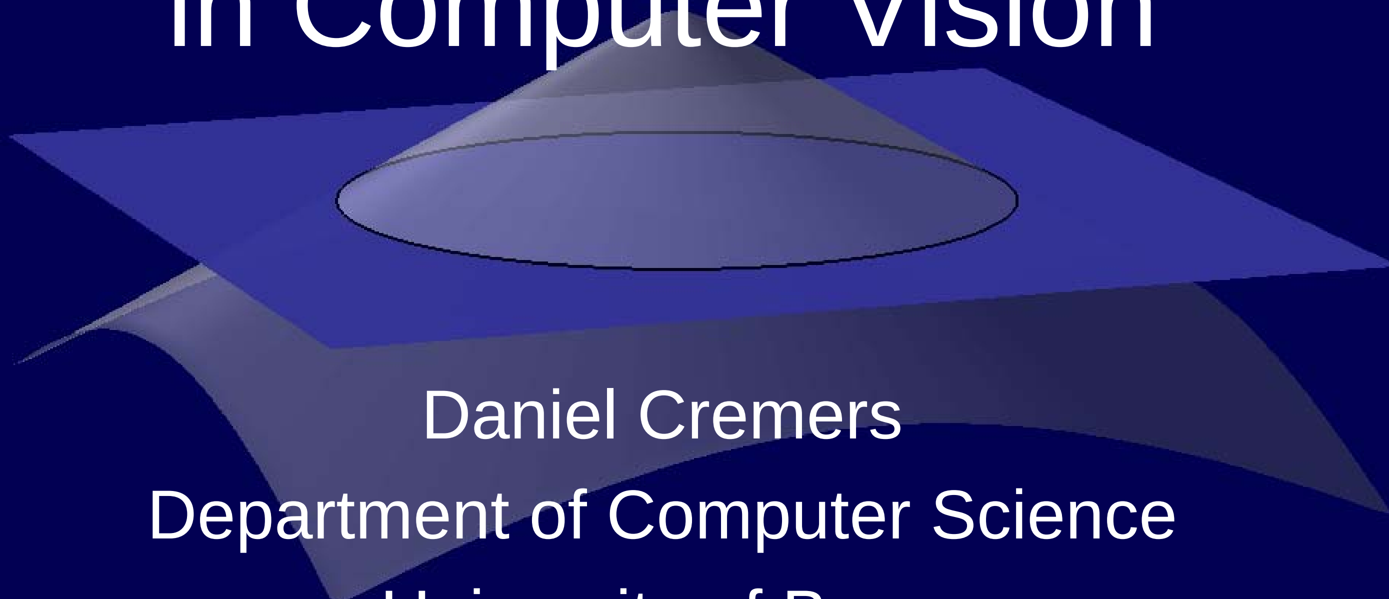
# Level Set Methods in Computer Vision



Daniel Cremers
Department of Computer Science
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Graz, May 7 2006

# Overview

Why level sets?  Explicit vs. implicit contours

Some examples: graphics & 3D reconstruction

Level set methods for image segmentation

Segmenting texture and motion information

Statistical shape priors for level set functions

*Cremers, Rousson, Deriche, IJCV 2006*
*"A Review of Statistical Approaches to Level Set Segmentation: Integrating color, texture, motion and shape"*

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# Continuous Space and Infinite-dimensional Optimization

Motion of a hypersurface

$$C \subset \mathbb{R}^n$$

Application domains:

- Computational physics
- Fluid mechanics
- Optimal design
- Computer Graphics
- Computer Vision
- ...

Evolution equation: $\frac{dC}{dt} = F\,\vec{n}$

Energy minimization: $E(C) \longrightarrow \min$

# Evolution of Explicit Boundaries

$$C : [0,1] \times [0,T] \rightarrow I\!R^2$$

$$C(s,t) = \sum_{j=1}^{n} x_j(t)\, B_j(s)$$

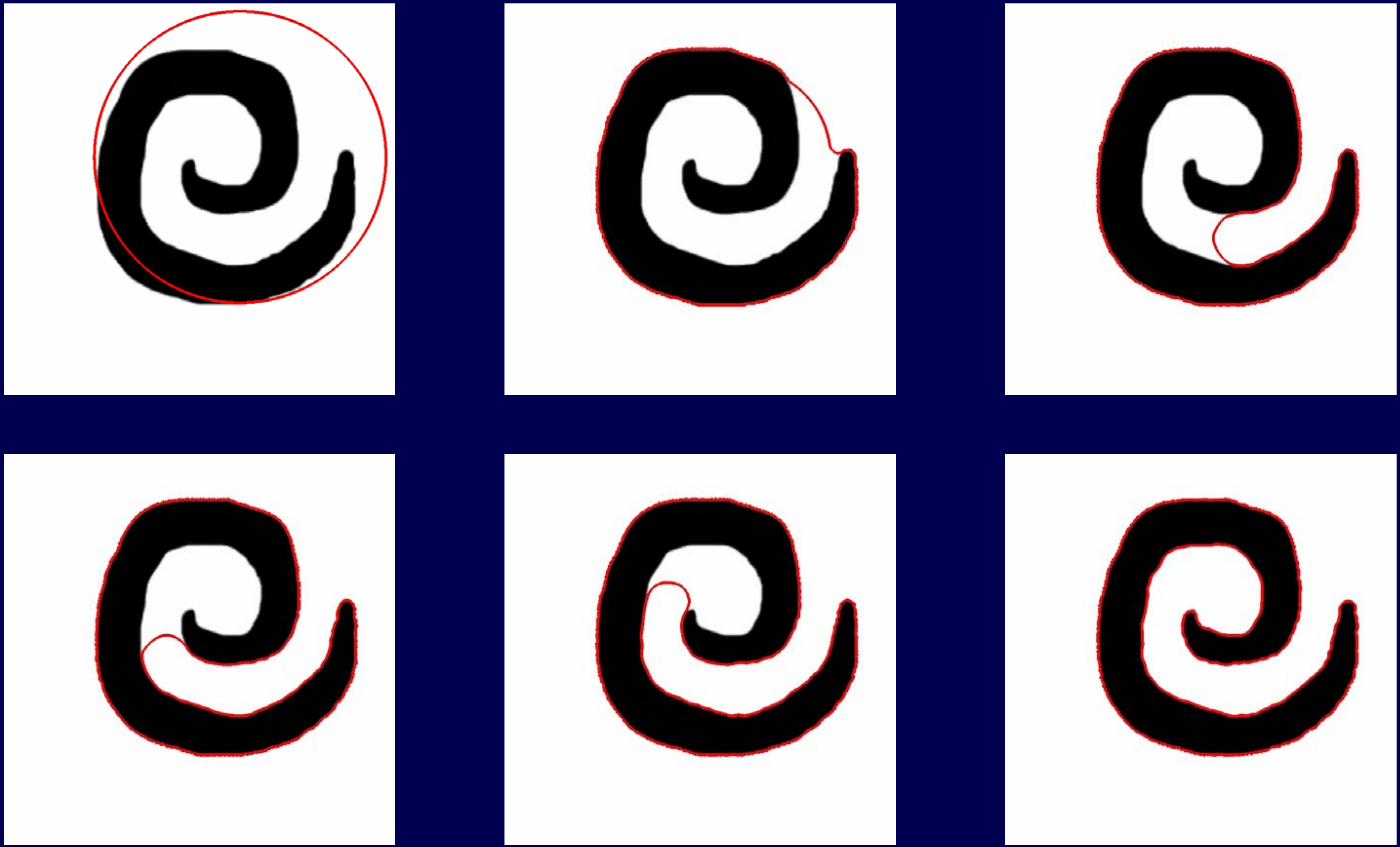
control points basis functions

$$\frac{\partial C}{\partial t} = \sum_j \dot{x}_j(t)\, B_j(s) = F\,\vec{n}$$

$$\sum_j \dot{x}_j(t)\, \underbrace{\langle B_i, B_j \rangle}_{\equiv B_{ij}} = \underbrace{\langle B_i, F\vec{n} \rangle}_{\equiv b_i} \;\Rightarrow\; \boxed{\dot{\vec{x}}(t) = B^{-1}\, b}$$

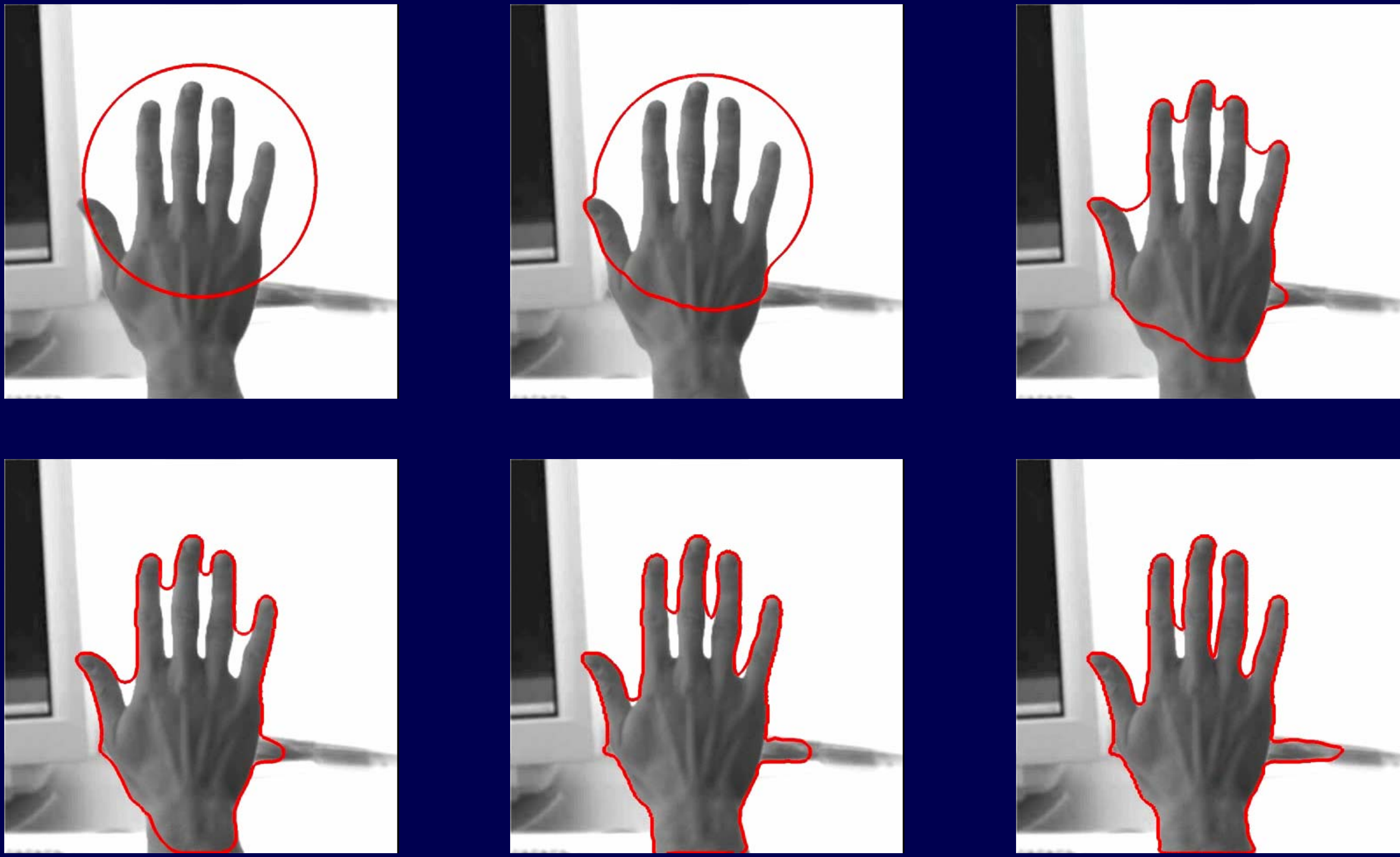
control point evolution

# Evolution of Explicit Boundaries



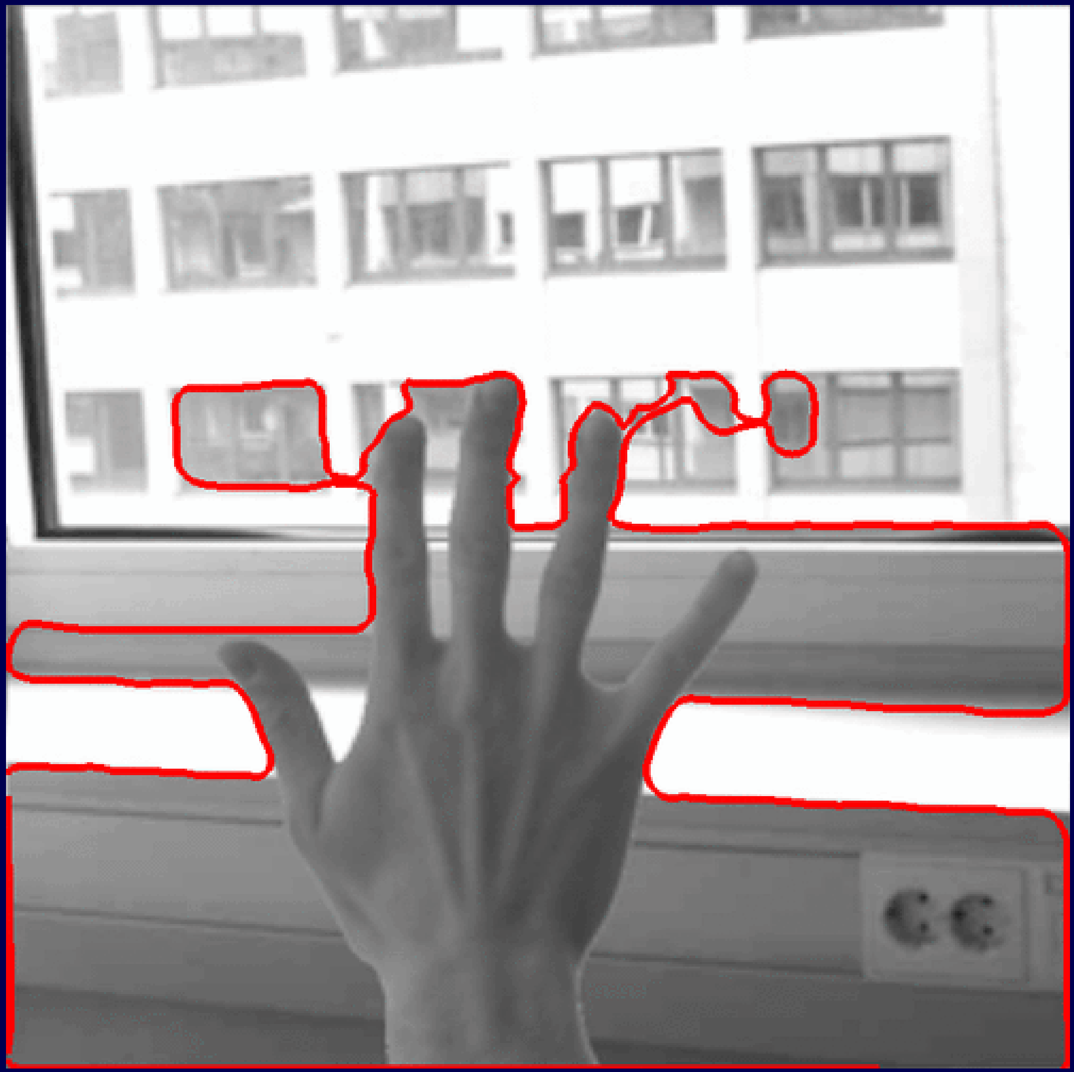
*Cremers, Tischhäuser, Weickert, Schnörr, "Diffusion Snakes", IJCV '02*

# Evolution of Explicit Boundaries



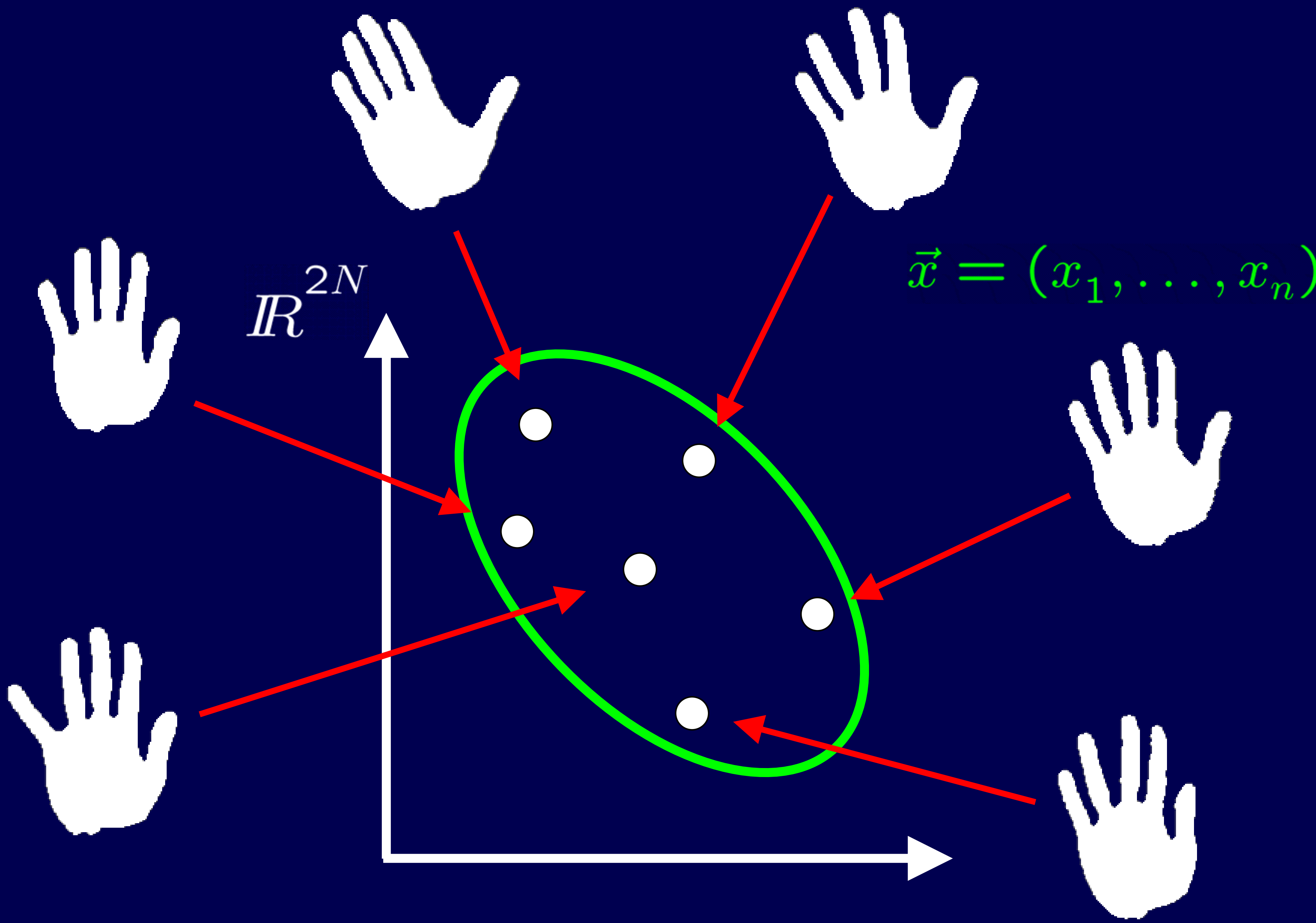
*Cremers, Tischhäuser, Weickert, Schnörr, "Diffusion Snakes", IJCV '02*

# Evolution of Explicit Boundaries

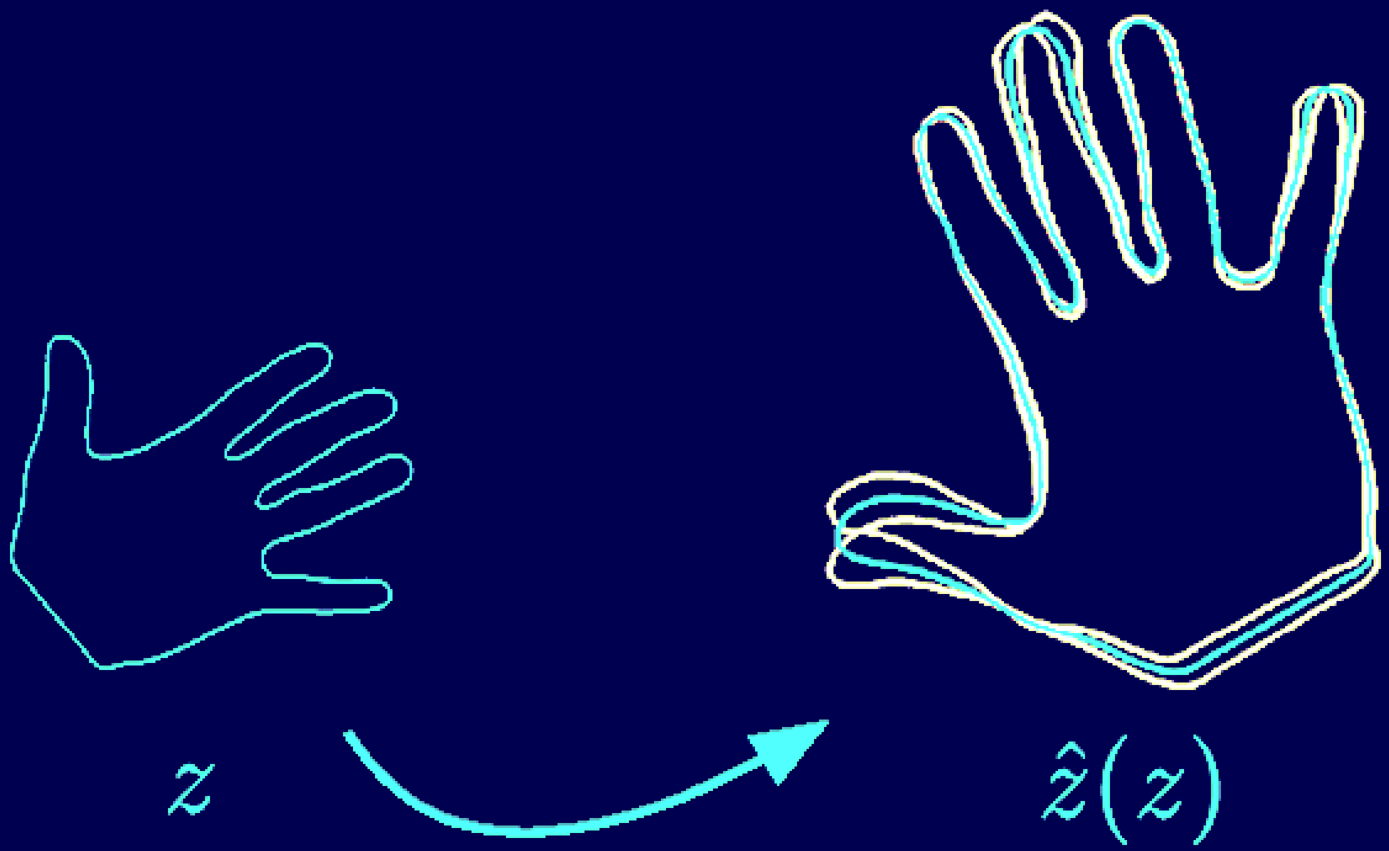


*Cremers, Tischhäuser, Weickert, Schnörr, "Diffusion Snakes", IJCV '02*

# Statistical Learning of Explicit Shapes



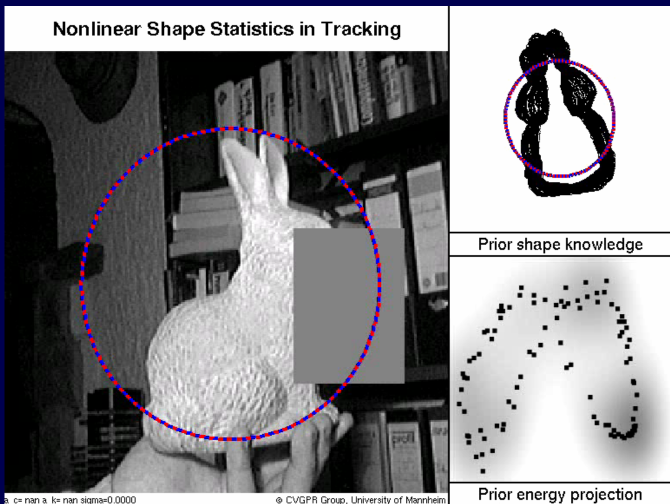
# Alignment of Explicit Contours



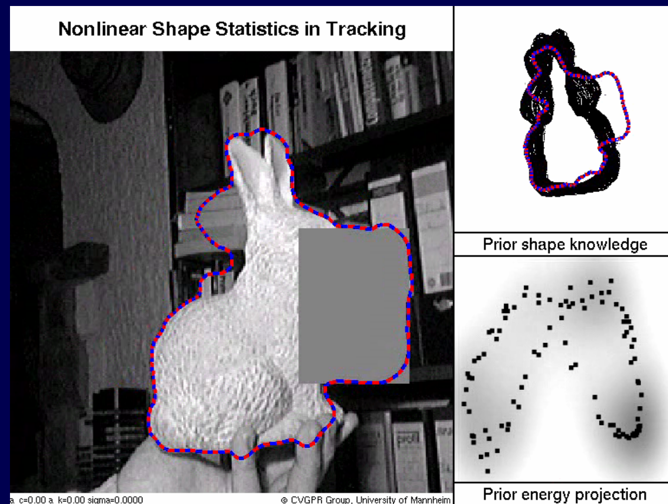
# Segmentation with Statistical Shape Prior

*Cremers, Tischhäuser, Weickert, Schnörr, "Diffusion Snakes", IJCV '02*

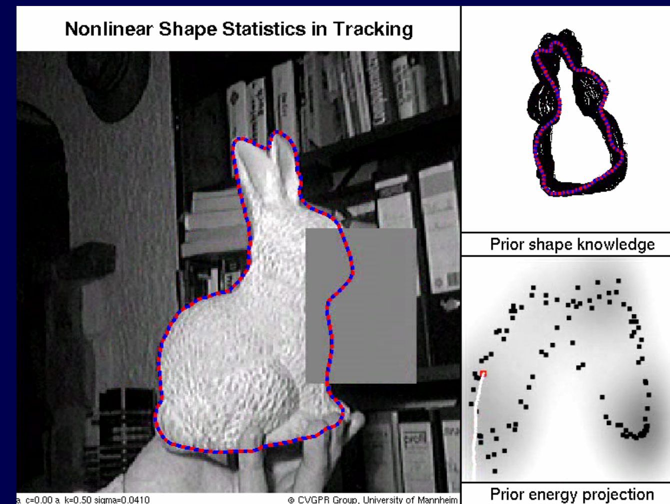
# Tracking with Kernel Shape Prior



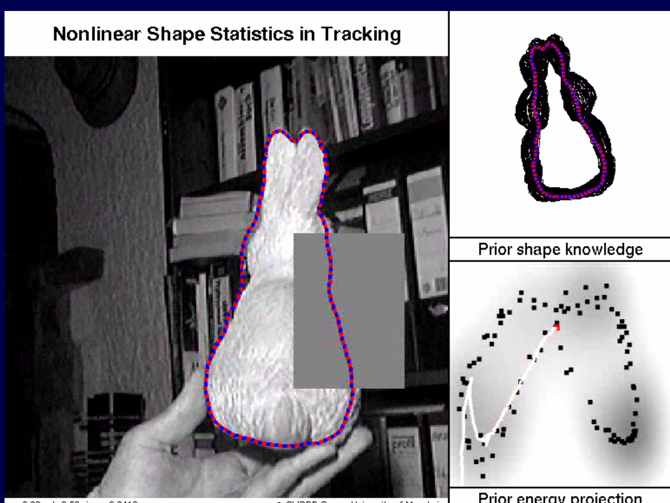

initial contour



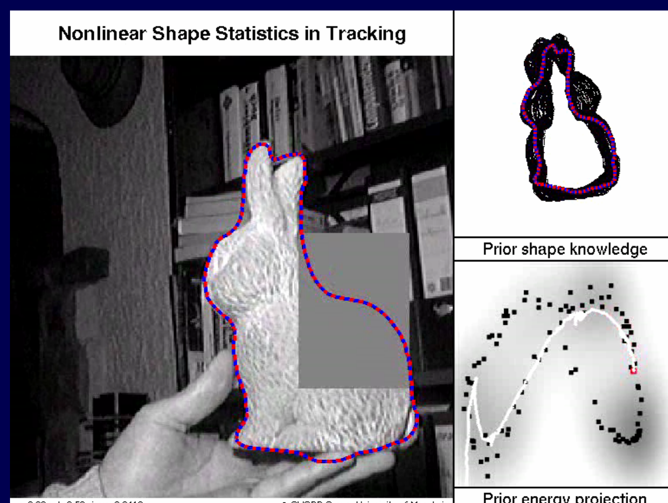

no prior



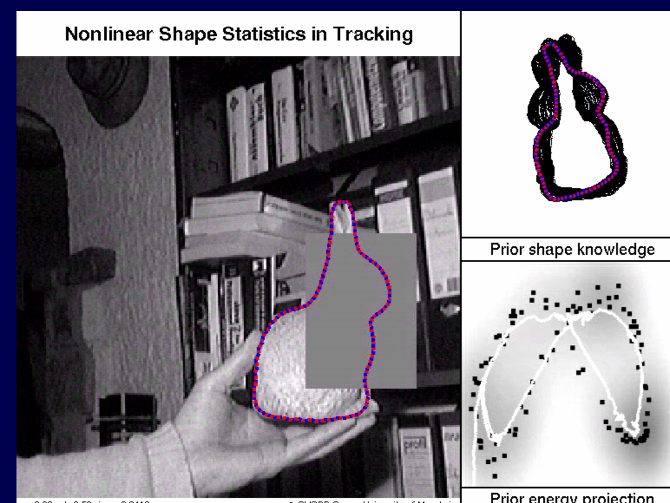

with prior




with prior




with prior

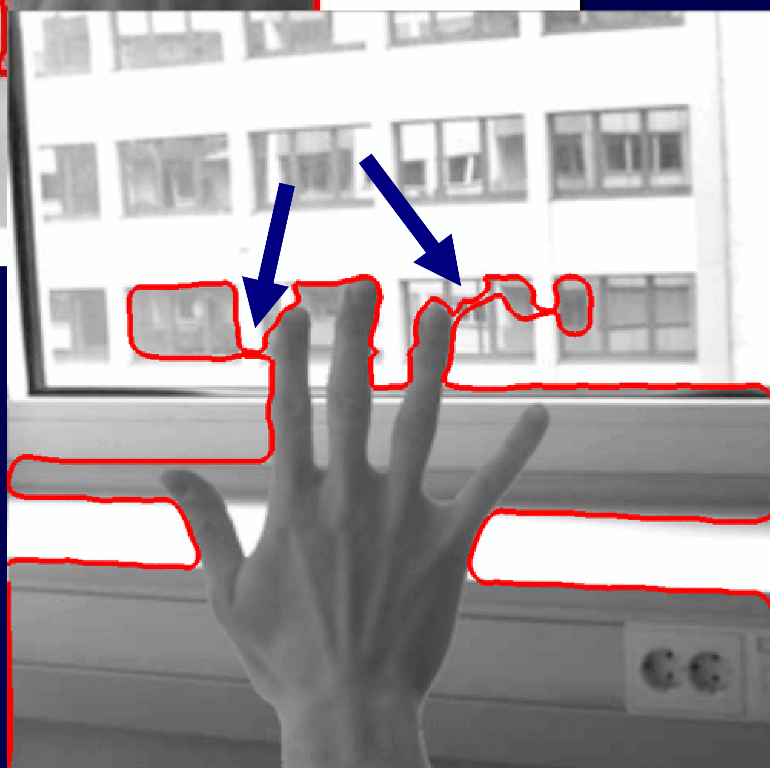



scale invariance

*Cremers, Kohlberger, Schnörr, ECCV 2002*

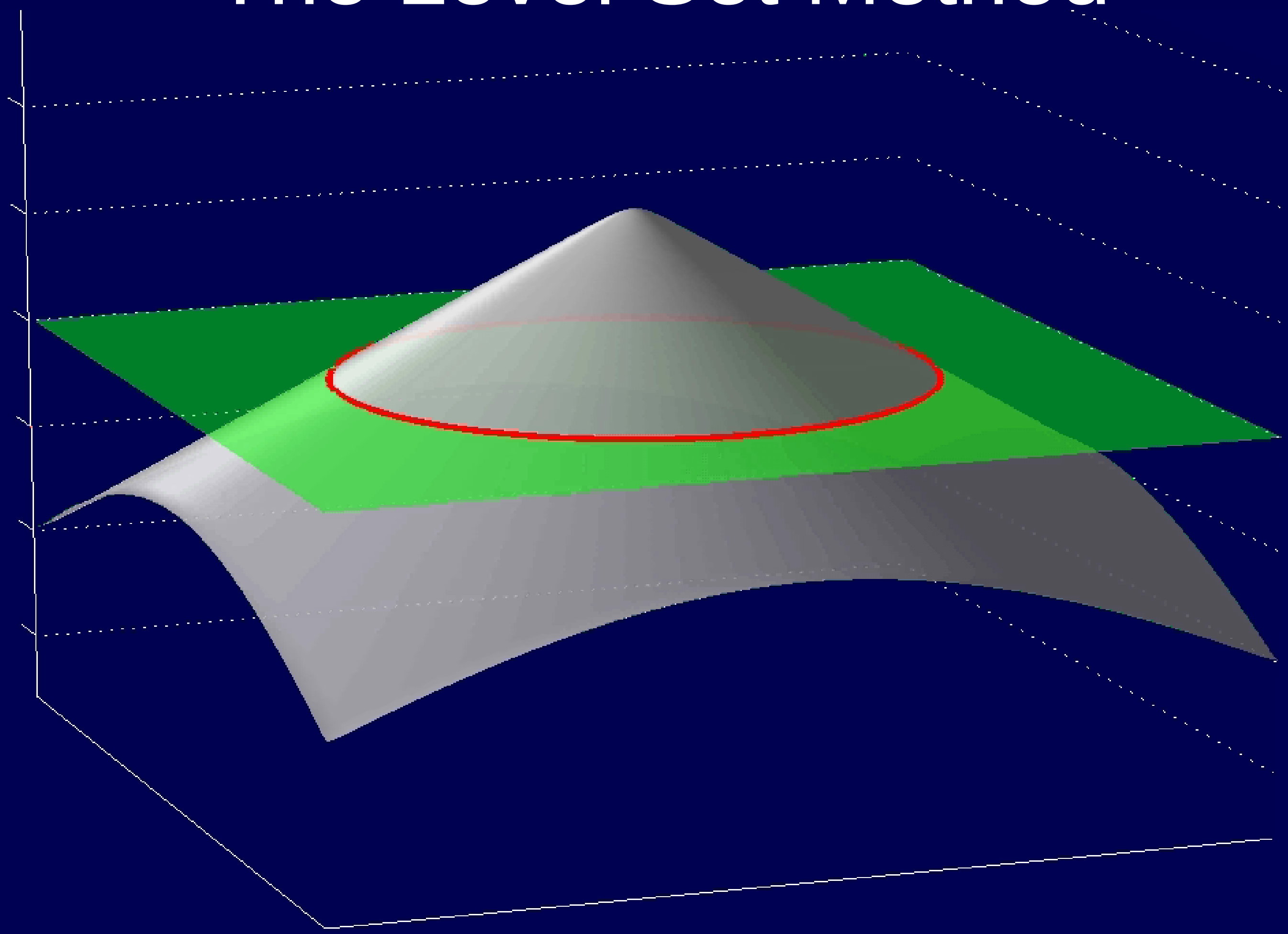
# Limitations of Explicit Representations

Insufficient resolution / control point density
requires control point regridding mechanisms



Fixed topology
requires heuristic splitting mechanisms

# The Level Set Method



$$C = \{x \in \Omega \,|\, \phi(x) = 0\}, \quad \phi : \Omega \subset \mathbb{R}^n \to \mathbb{R}$$

*Osher, Sethian, J. of Comp. Phys. '88*

*Dervieux, Thomasset, '79, '81*

# The Level Set Method

Assume the interface evolves according to: $\frac{dC}{dt} = F\,\vec{n}$

At all times the interface is the zero level of $\phi$ :

$$\phi\big(C(t),t\big) = 0 \quad \forall\, t.$$

Then the total time derivative of must vanish:

$$0 = \frac{d}{dt}\phi\big(C(t),t\big) = \nabla\phi\,\frac{dC}{dt} + \partial_t\phi.$$

We obtain an evolution equation for $\phi$ : $\partial_t\phi = -\nabla\phi\,\frac{dC}{dt} = -\nabla\phi\,F\vec{n}.$

Using $\vec{n} = \frac{\nabla\phi}{|\nabla\phi|}$ , we obtain the level set equation:

$$\partial_t\phi = -F\,|\nabla\phi|.$$

# The Level Set Method

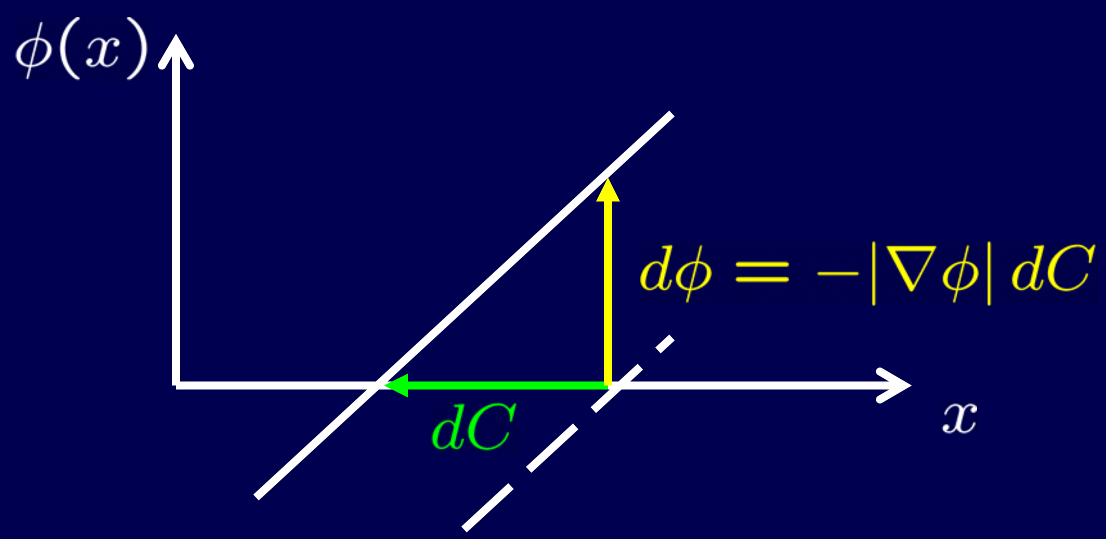
Thus the contour evolution

$$\frac{dC}{dt} = F\,\vec{n}$$

corresponds to an evolution of $\phi$ given by:

$$\partial_t \phi = -F\,|\nabla \phi|.$$

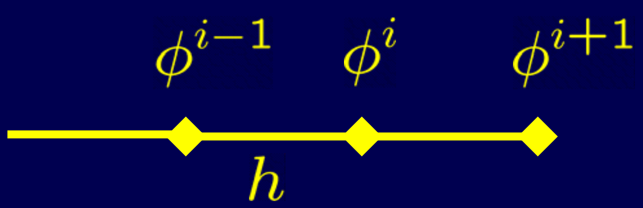
The scaling by $-|\nabla \phi|$ is easily verified in one dimension:



# Some major challenges

What are meaningful choices for the speed function $F$?
- interior versus exterior forces: liquids, gases, compressibility, viscosity, conservation laws, ...

How should one discretize respective quantities?
- choice of grid: Cartesian, unstructured, adaptive, ...
- symmetric differences, upwind schemes, stencils, ...




$$\phi_x^i \approx \phi^{i+1} - \phi^i, \qquad \phi_x^i \approx \frac{1}{2}\left(\phi^{i+1} - \phi^{i-1}\right)$$

What is the order of convergence upon refinement of the grid?

Level set methods in computer vision:
- Image segmentation, tracking, statistical shape modeling, multiview reconstruction, ...

# Efficient Implementations

There are numerous methods to speed up level set methods. In combination these lead to segmentation speeds close to real time for volumetric data of 512 x 512 x 64 voxels.

Popular methods include:

1. Narrow band methods - Evolve level set function around zero level: *Adalsteinsson, Sethian '95*

2. Multiresolution implementation - coarse-to-fine schemes.

3. Implicit discretization schemes, additive operator schemes: *Weickert '00, Goldenberg et al. '01*

4. Implementations on Graphics Hardware: *Rumpf, Strzodka '01, Lefohn et al. '03*

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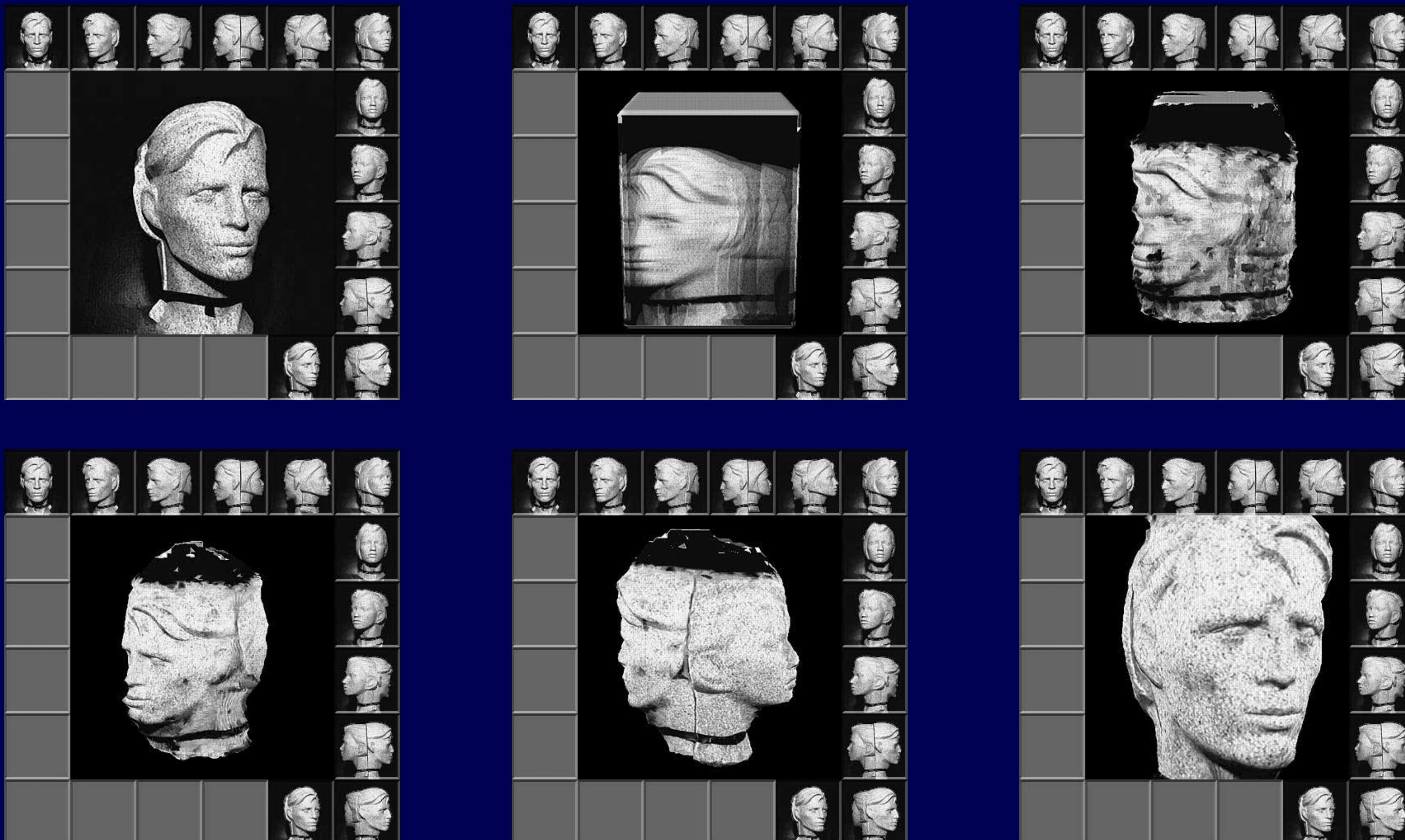
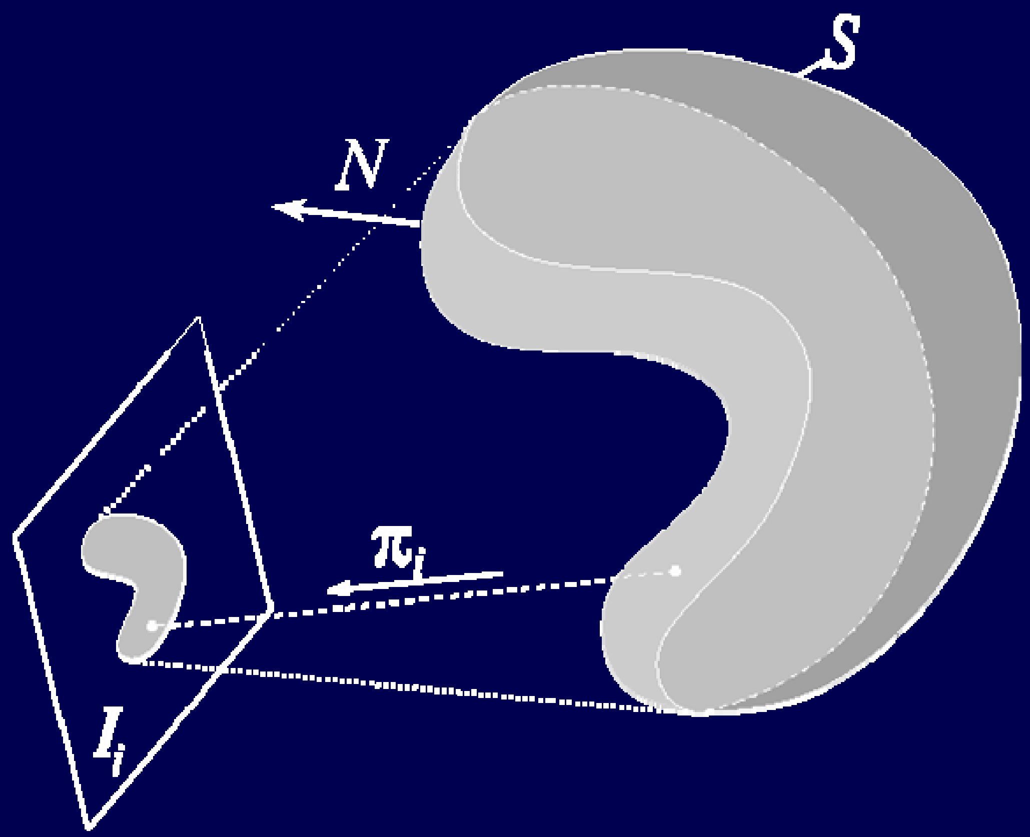
# Multiview Reconstruction with Level Sets



*Keriven, Faugeras '98:* based on matching via cross correlation

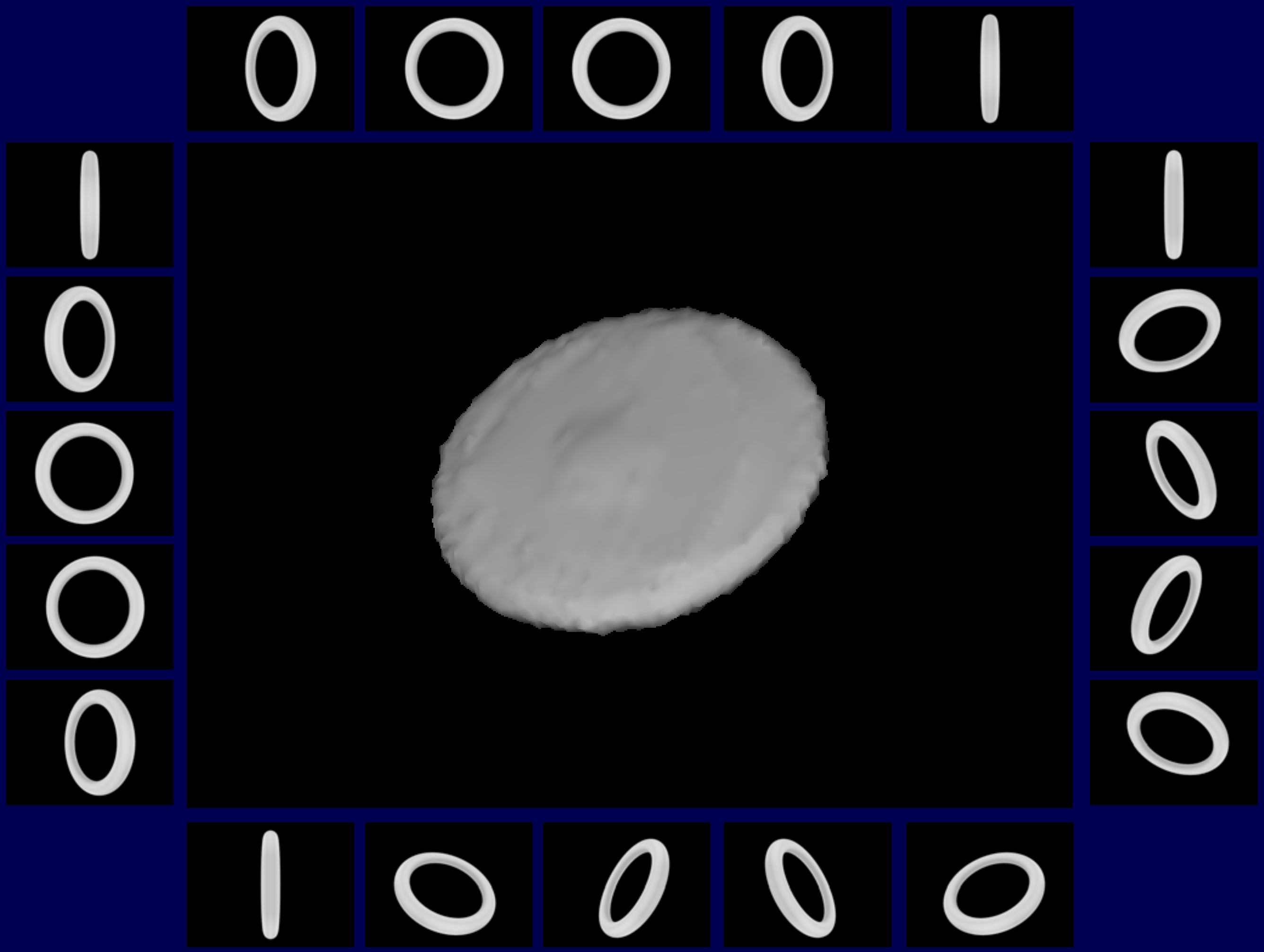
# Multiview Reconstruction with Level Sets




*Yezzi, Soatto, Stereoscopic Segmentation, IJCV '03*

"3D Segmentation": Jointly estimate shape and intensity of object and background

# Multiview Reconstruction with Level Sets



*Stereoscopic Segmentation*

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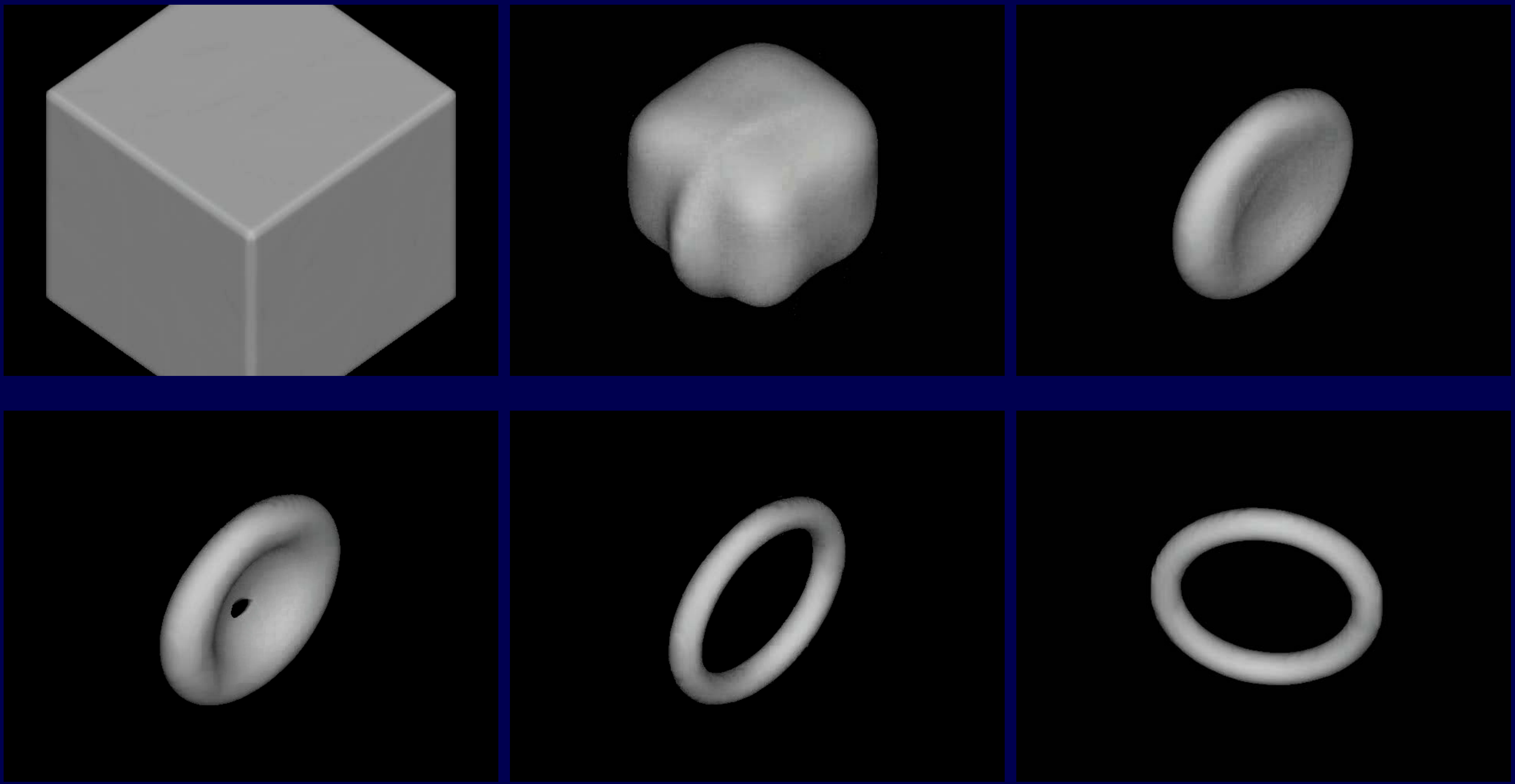

*Kolev, Brox, Cremers '06: Probabilistic Formulation*

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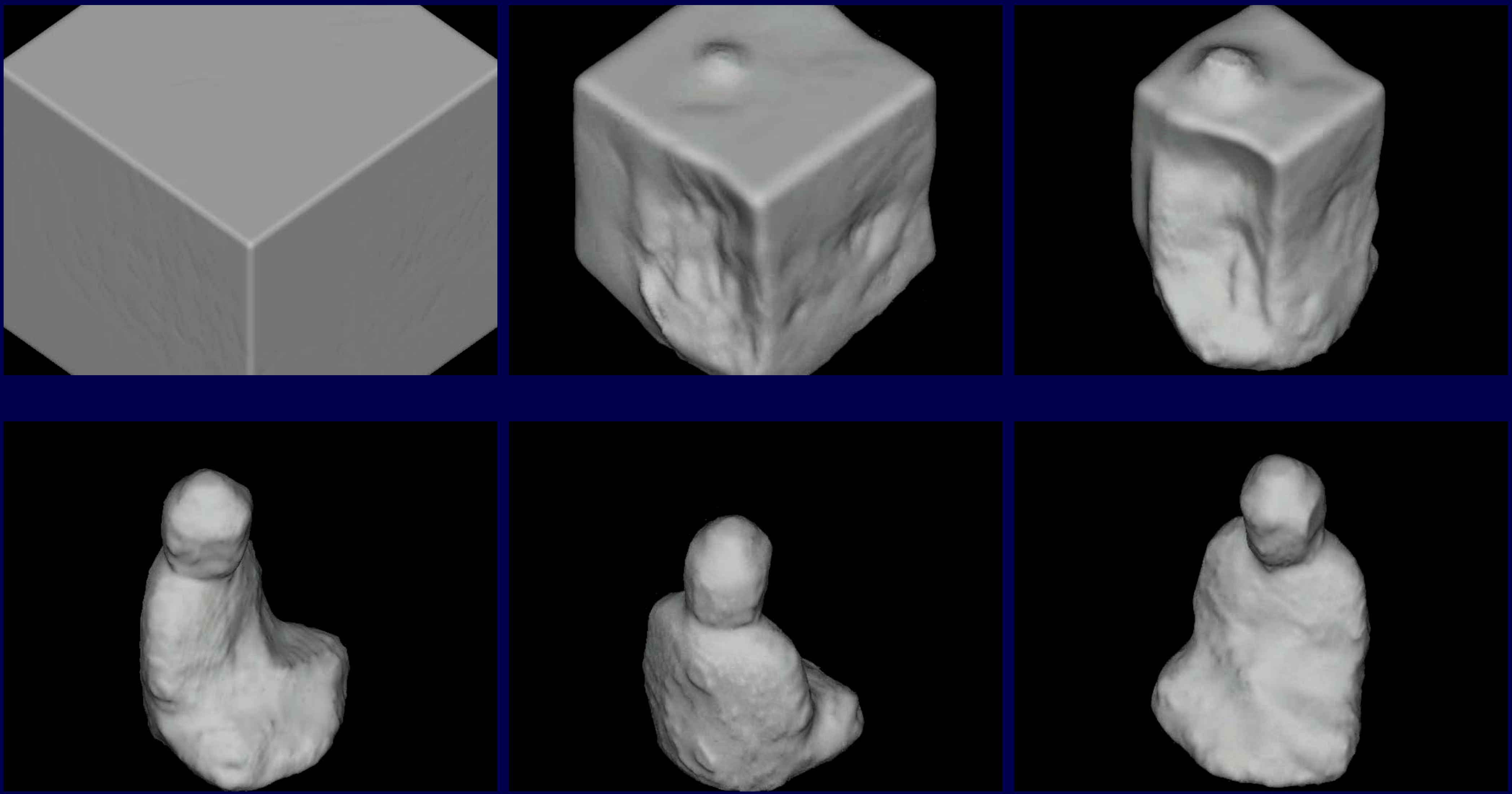

*Kolev, Brox, Cremers '06*

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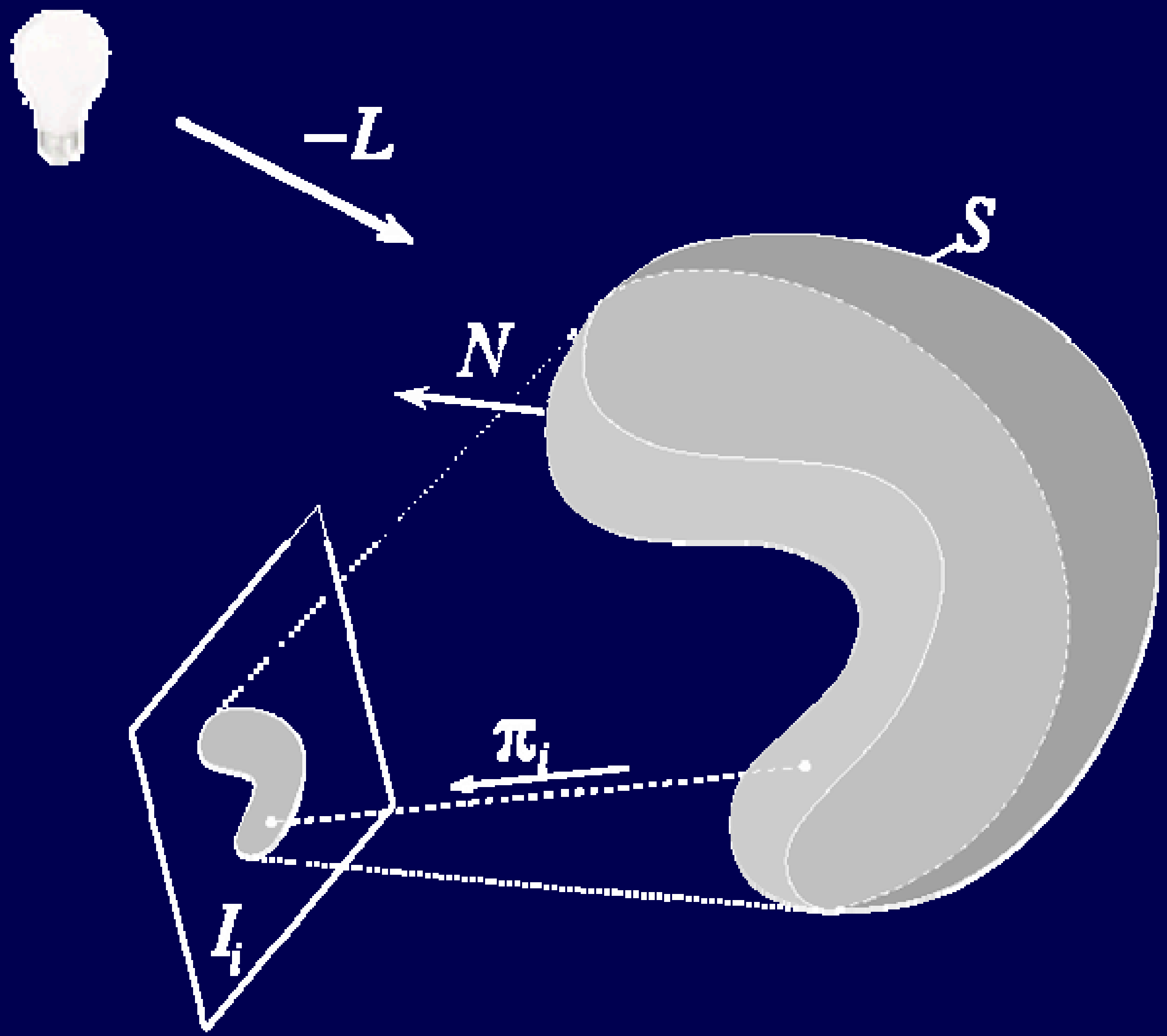
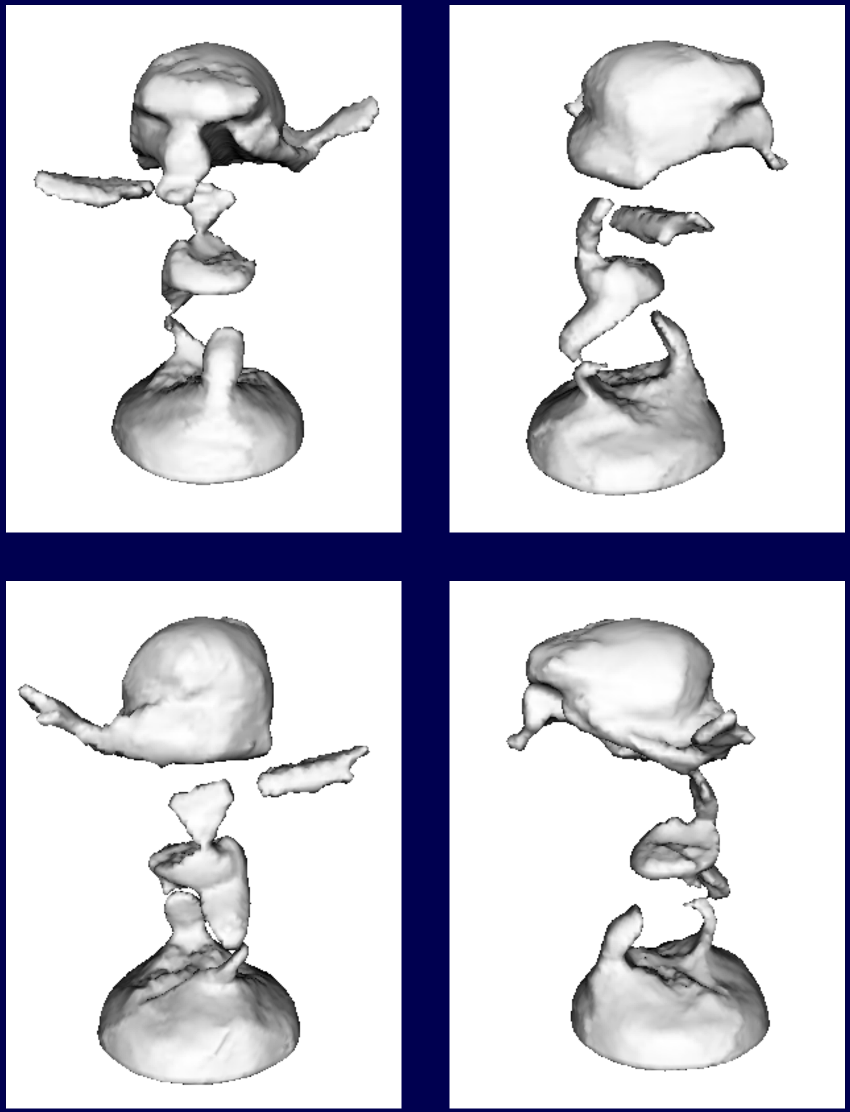



*Jin, Cremers,Yezzi, Soatto, CVPR '04:*
*Sheding Light on Stereoscopic Segmentation*

# Multiview Reconstruction with Level Sets



*Stereoscopic Segmentation*

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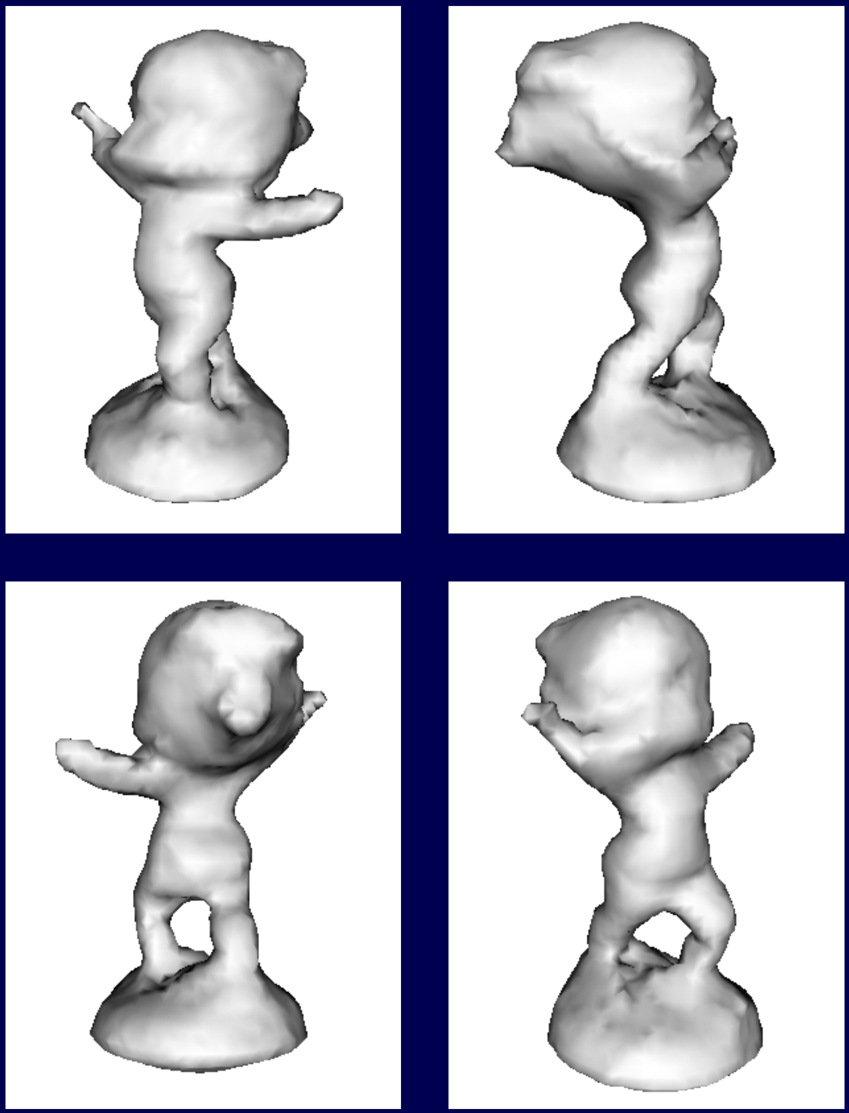

*Sheding Light on Stereoscopic Segmentation*

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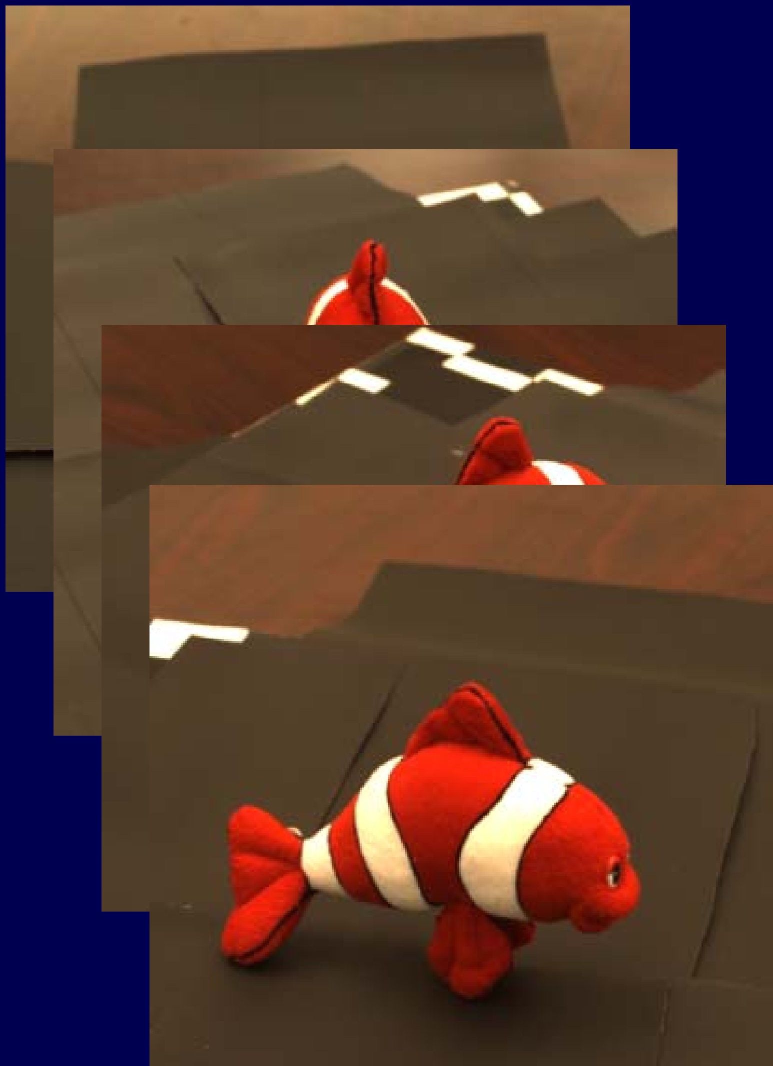

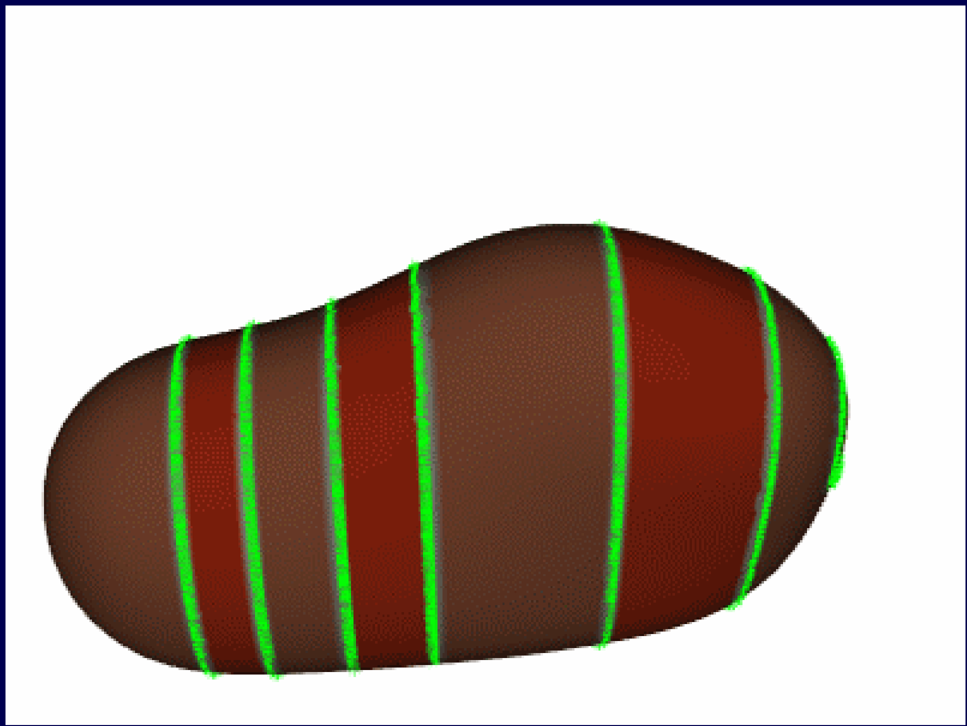

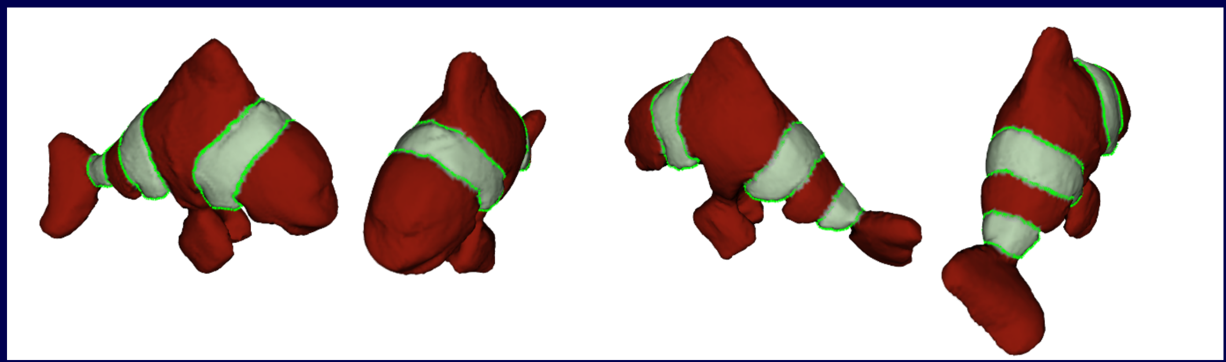

*Jin, Yezzi, Soatto, ECCV '04:*
dynamical evolution of surface and contours

# Multiview Reconstruction with Level Sets

*Labatut, Keriven, Pons '06:* Based on image-to-image matching and cross correlation, one of most accurate and fastest, GPU implementation takes ~ 240 s (see comparison in CVPR '06)

# Some Related Work

## The Level Set Method

Dervieux, Thomasset ’79, ’81
Osher, Sethian ’88
Sethian ’96, ’99
Osher, Fedkiw ’02
Osher, Paragios ‘03

## Level Sets for 3D Reconstruction

Keriven, Faugeras ’98
Yezzi, Soatto ’03
Goldluecke, Magnor ’04
Jin, Cremers, Yezzi, Soatto ’04
Pons, Keriven, Faugeras ’05
Kolev, Brox, Cremers ’06

## Level Sets for Segmentation

Caselles et al. ’93
Malladi, Sethian, Vemuri ’95
Caselles, Kimmel, Sapiro ’95
Kichenassamy et al. ’95
Paragios, Deriche ’00
Chan, Vese ’01, Tsai, Yezzi, Willsky ’01
Heiler, Schnoerr ’03
Cremers, Soatto ’05
Brox, Weickert ’06

## Shape Knowledge for Level Sets

Leventon, Grimson, Faugeras ’00
Tsai et al. ’01
Rousson, Paragios ’02
Rousson, Paragios, Deriche ’03
Charpiat, Faugeras, Keriven ’03
Cremers, Sochen, Schnörr ’03
Riklin-Raviv, Sochen, Kiryati ’04
Rathi, Vasvani et al. ’05
Cremers, Osher, Soatto ’06

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# Image Segmentation: Edge-based

*Kass, Witkin, Terzopoulos, "Snakes" '88:*

$$E(C) = -\underbrace{\int |\nabla I(C)|^2 ds}_{\text{external energy}} + \underbrace{\int \left\{ \nu_1 |C_s|^2 + \nu_2 |C_{ss}|^2 \right\} ds}_{\text{internal energy}}$$

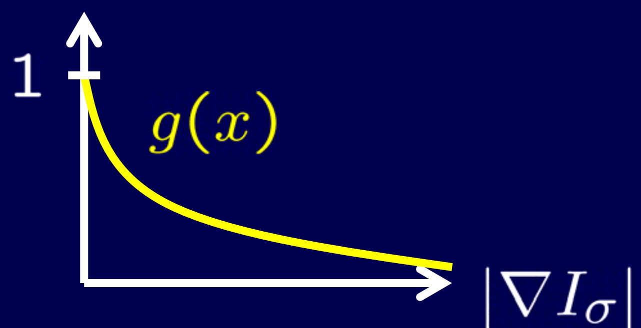
Image $I : \Omega \to \mathbb{R}$, parametric contour $C : [0,1] \to \Omega$

*Caselles et al.'93, Caselles et al.'95, Kichenassamy et al. '95:*

$$E(C) = \int g(C(s))\, ds \qquad g(x) = \frac{1}{1 + |\nabla I_\sigma(x)|^2}$$

edge indicator function ($g(x)$)

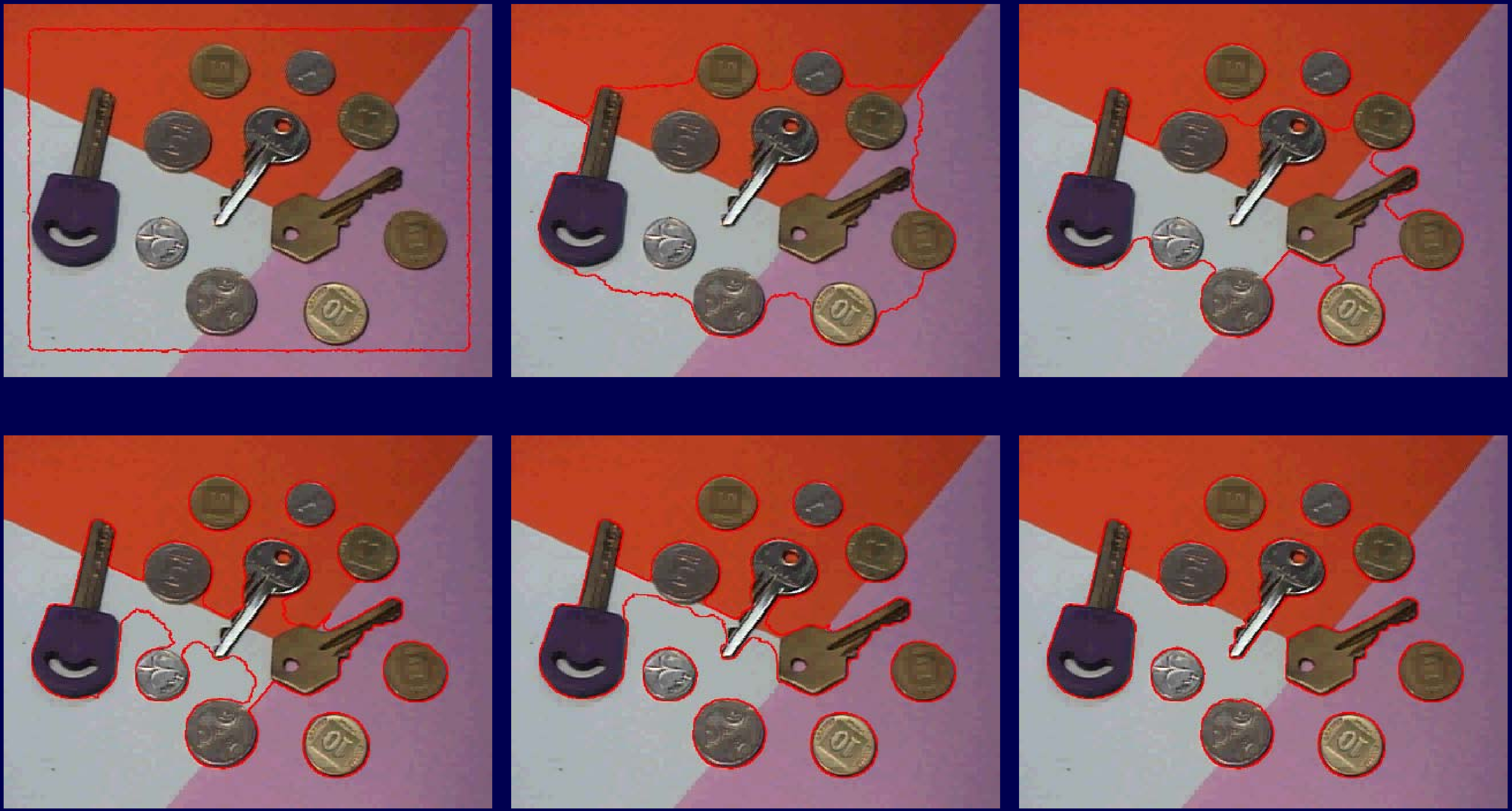
smoothed image ($I_\sigma$)




*geodesic active contours:*

$$\frac{\partial \phi}{\partial t} = |\nabla \phi| \, \text{div} \left( g(x) \frac{\nabla \phi}{|\nabla \phi|} \right)$$

# Image Segmentation: Edge-based



*Goldenberg, Kimmel, Rivlin, Rudzsky, IEEE TIP '01*

# Image Segmentation: Region-based

$$E(u,K) = \int_{\Omega} (I-u)^2 dx + \lambda \int_{\Omega \setminus K} |\nabla u|^2 dx + \nu_o \mathcal{H}^1(K)$$

*Mumford, Shah '85, '89*
*Blake, Zisserman '87*

$\Omega \subset \mathbb{R}^2$ Image domain

$I : \Omega \to \mathbb{R}$ Input image

$u : \Omega \to \mathbb{R}$ Segmented image

$K \subset \Omega$ Discontinuity set

$\lambda \to \infty$ : piecewise constant model

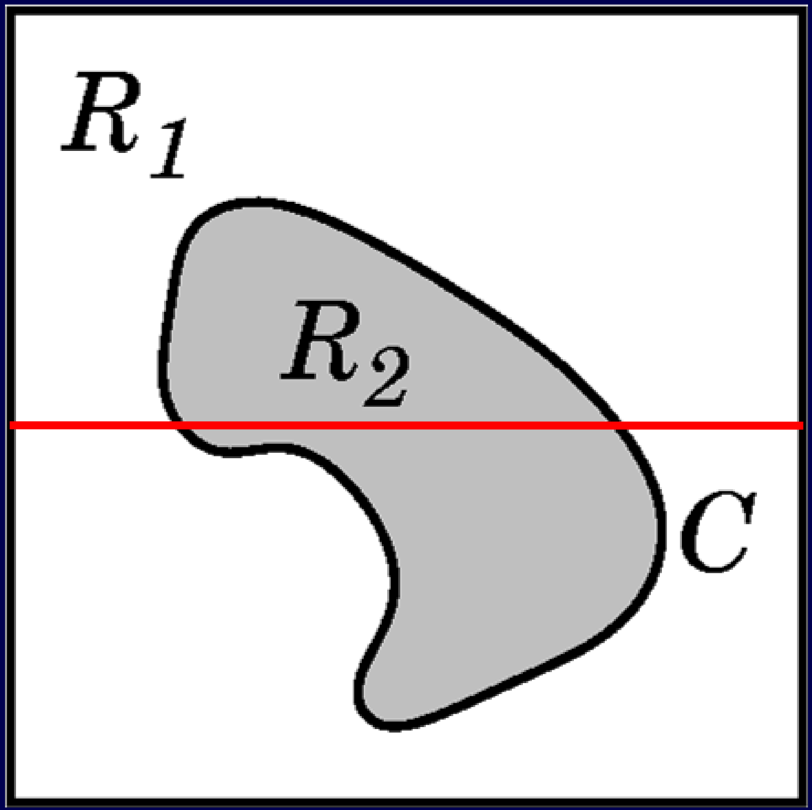
$$E(u,K) = \sum_i \int_{R_i} \left(I(x) - u_i\right)^2 dx \; + \; \nu \, |K|$$

*Mumford, Shah '89*
spatially discrete: *Blake '83*

# Image Segmentation: Region-based

$$E(u,K) = \sum_i \int_{R_i} \left(I(x) - u_i\right)^2 dx \; + \; \nu \, |K|$$

piecewise constant model



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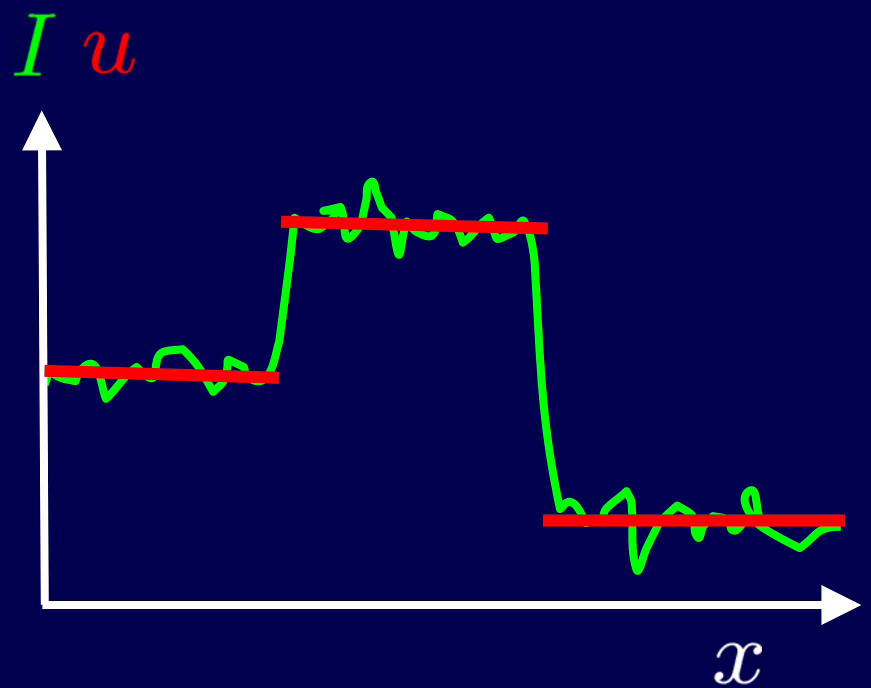
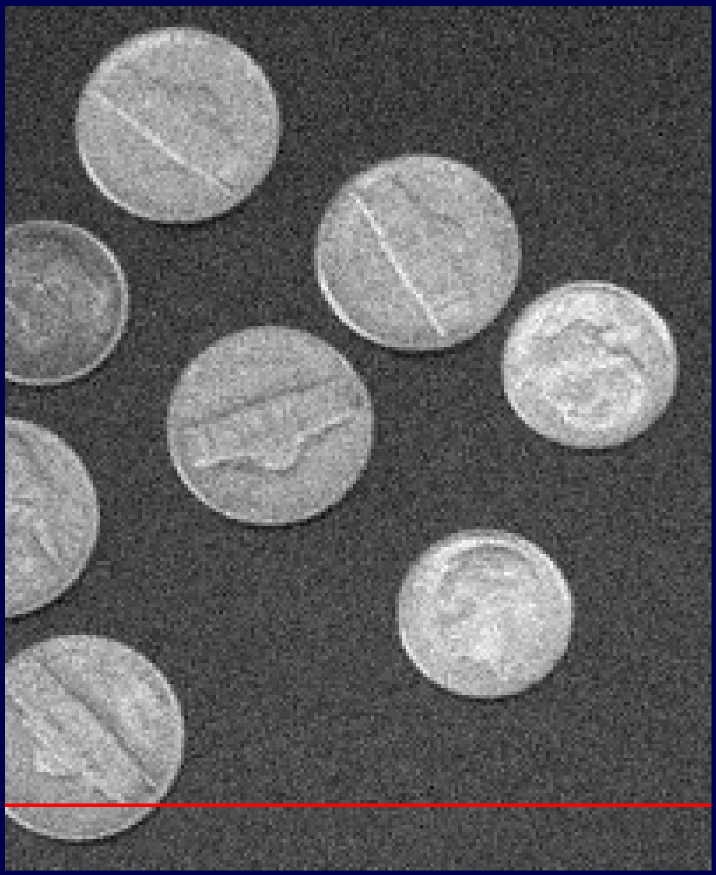
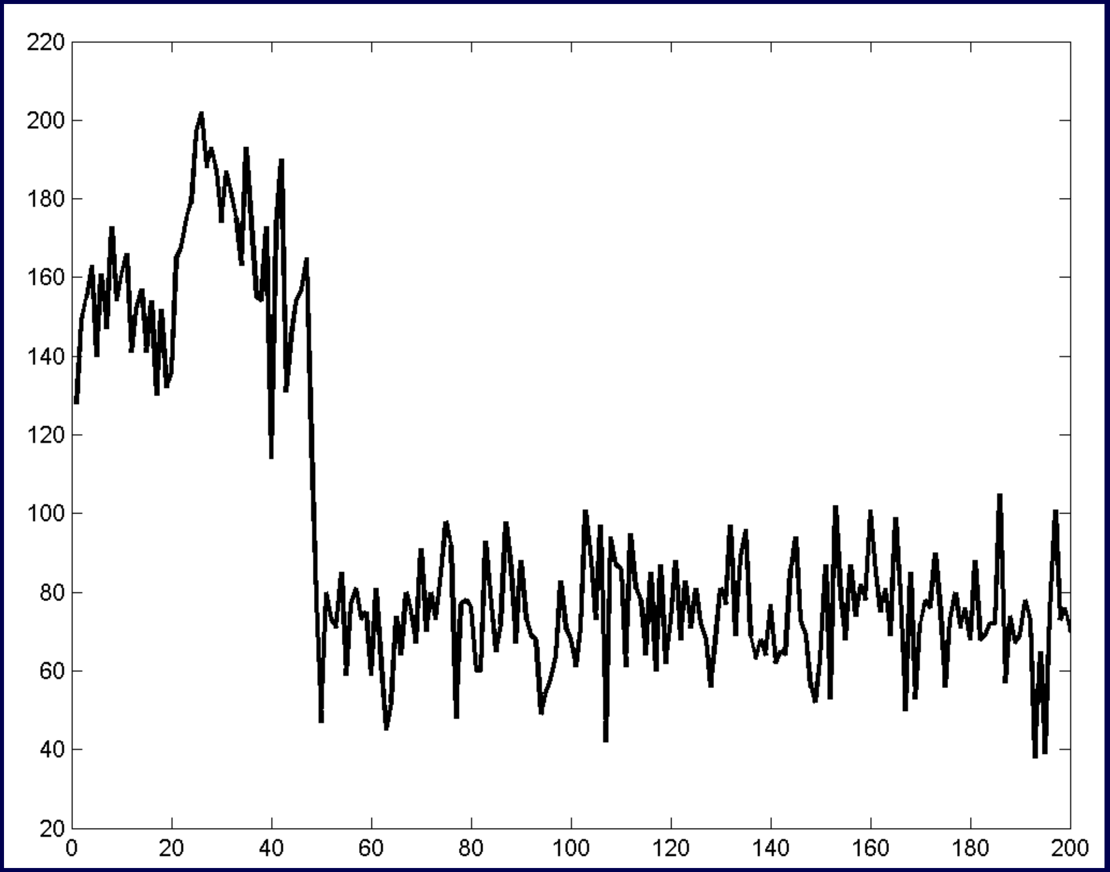


# Quantitative Comparison on 1-D



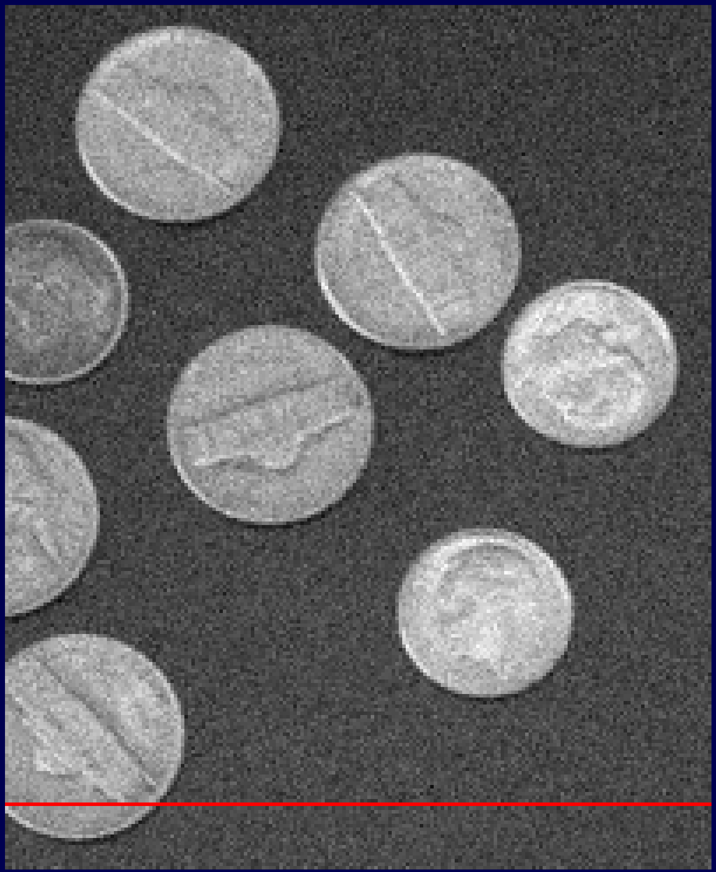
Input image



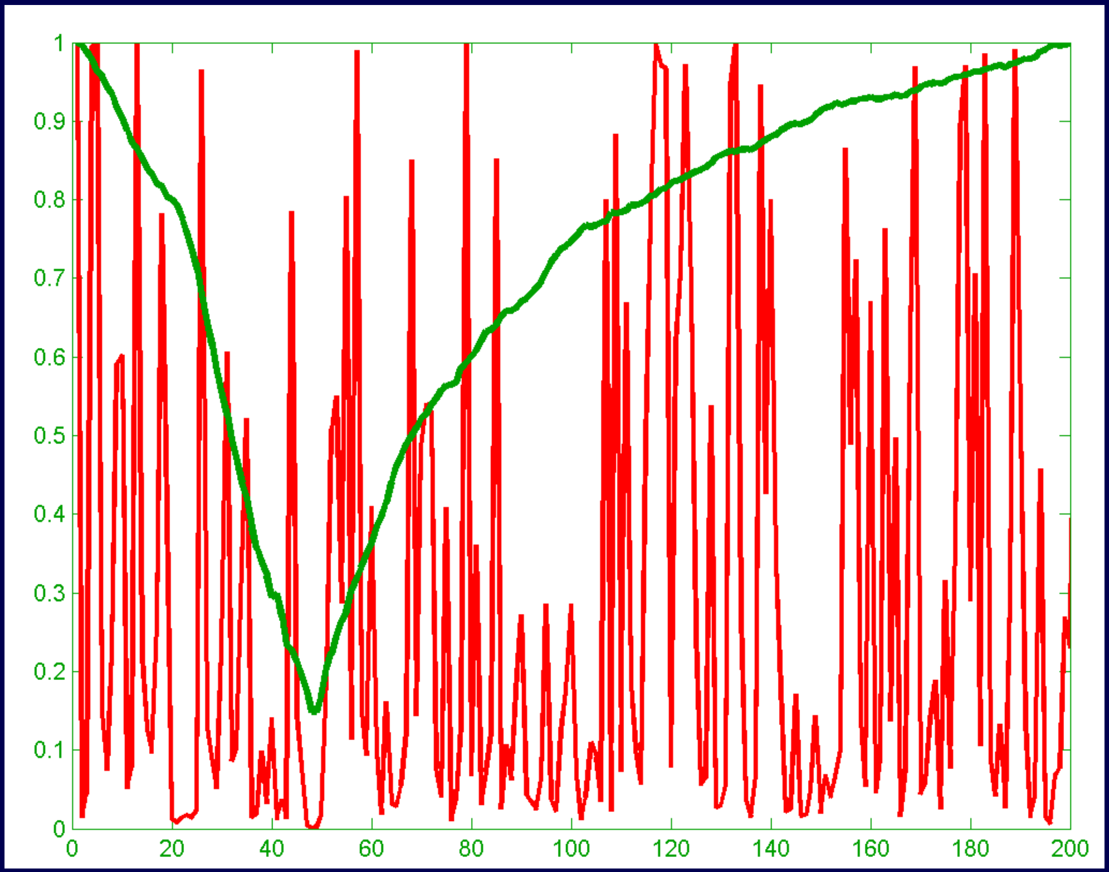

Intensity along 1D slice

*Cremers, Rousson, Deriche, IJCV ’06*

# Quantitative Comparison on 1-D



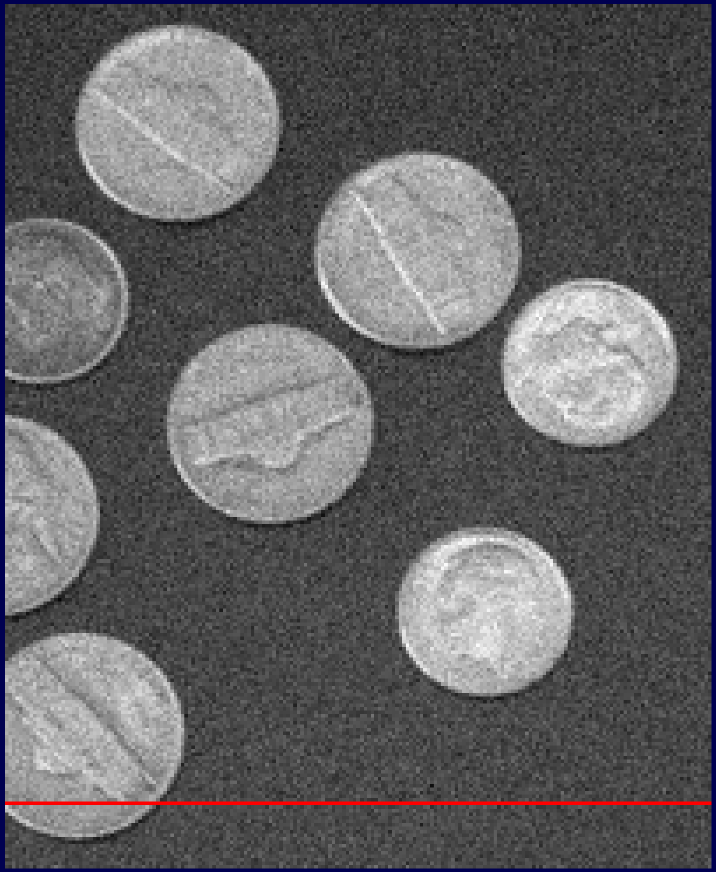
Input image



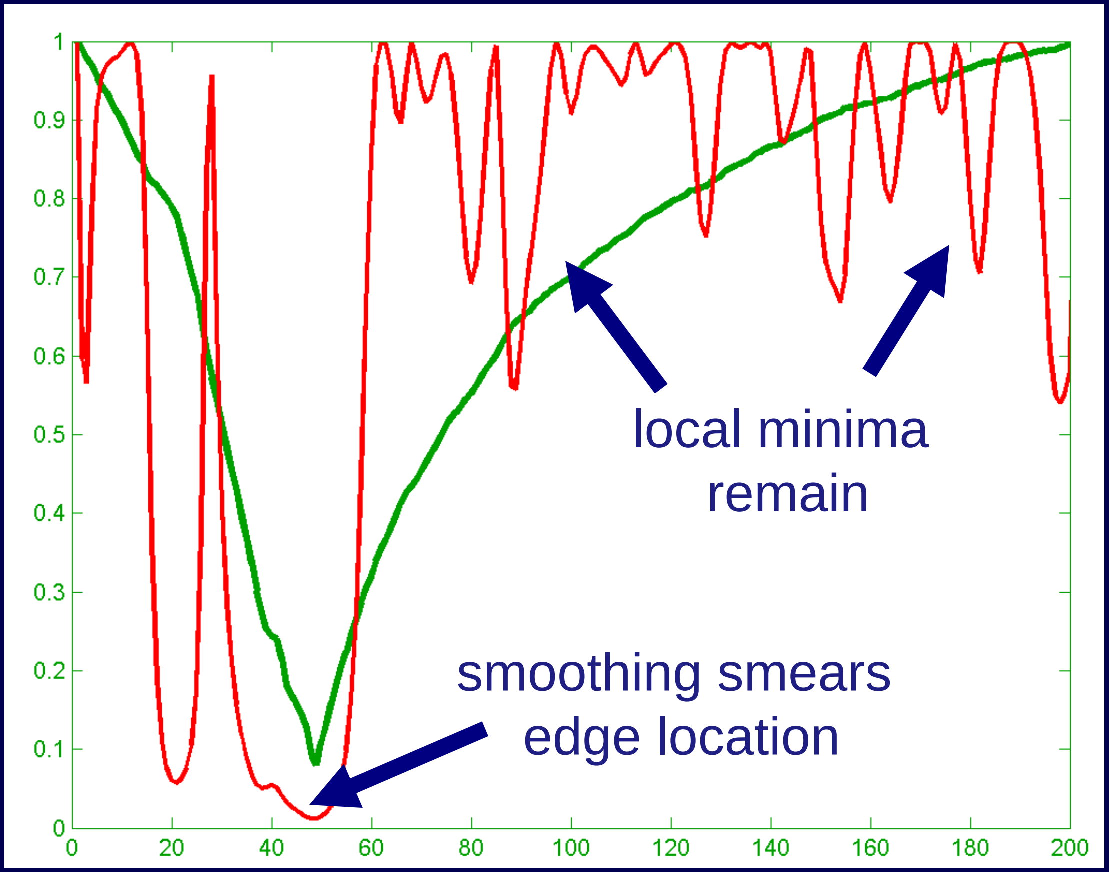

Energy of 1D segmentation for edge-based and region-based energies

*Cremers, Rousson, Deriche, IJCV '06*

# Quantitative Comparison on 1-D



Input image



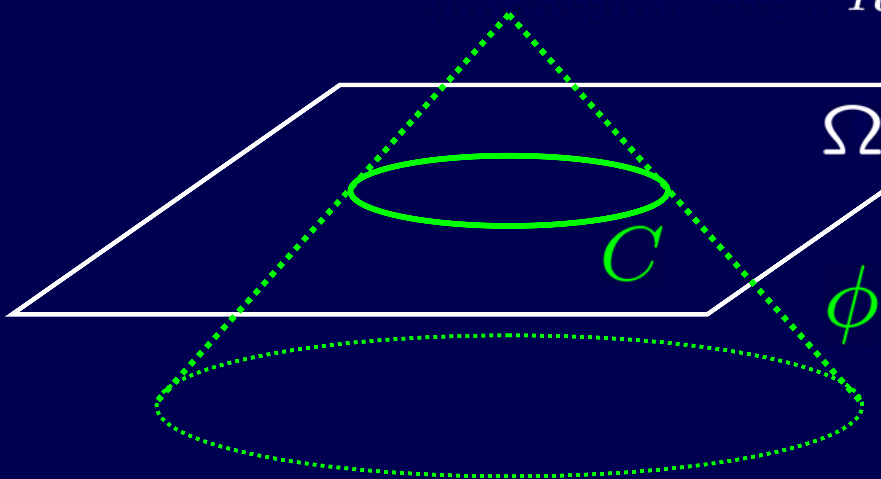

Energy for region-based and edge-based after smoothing

*Cremers, Rousson, Deriche, IJCV '06*

# Level Set Formulation of Mumford-Shah

Chan, Vese '99, Tsai et al. '00

$$E(u,C) = \sum_i \int_{R_i} \big(I(x) - u_i\big)^2 dx + \nu\,|C|$$




$$H\phi \equiv H(\phi) = \begin{cases} 1, & \text{if } \phi > 0 \\ 0, & \text{else} \end{cases}$$

Use smoothed step function

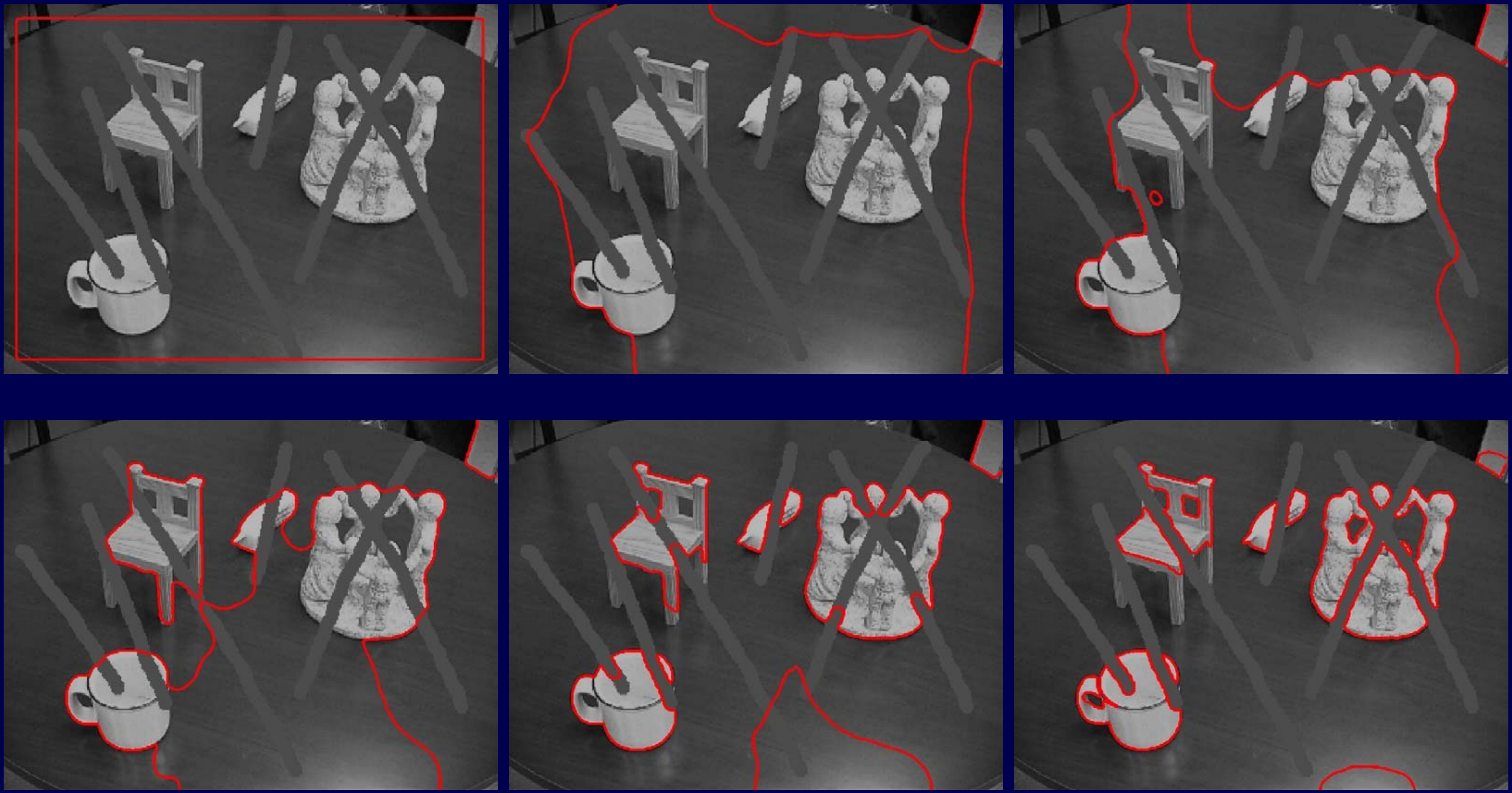
$$E(\phi,u) = \int_\Omega (I-u_1)^2 H\phi + (I-u_2)^2 \big(1-H\phi\big)\, dx + \nu \int_\Omega |\nabla H\phi| dx$$

$$\frac{\partial\phi}{\partial t} = -\frac{\partial E}{\partial \phi} = \delta(\phi)\left(\nu\,\text{div}\left(\frac{\nabla\phi}{|\nabla\phi|}\right) + (I-u_2)^2 - (I-u_1)^2\right)$$

# Level Set Formulation of Mumford-Shah

*Chan & Vese '99 , Implementation: D. Cremers*

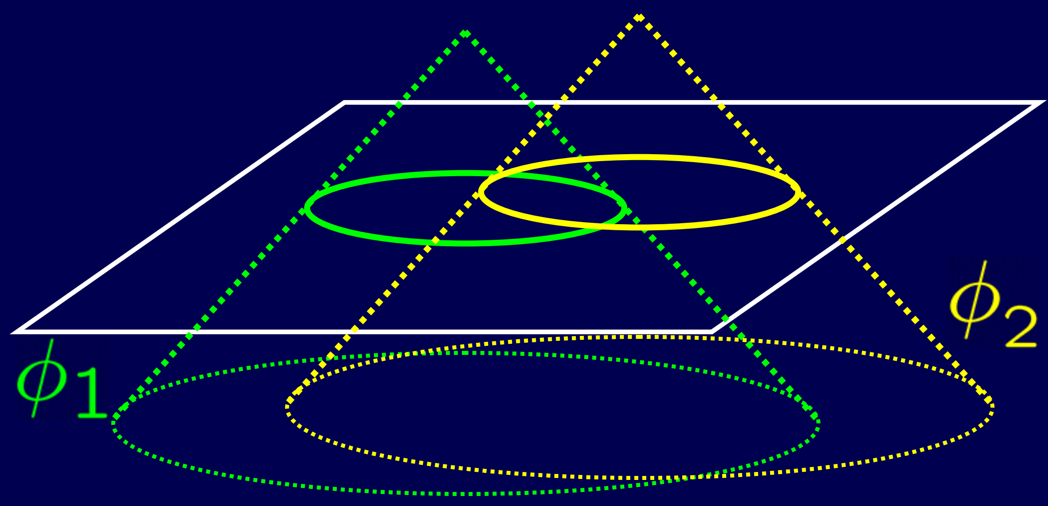
# Level Set Formulation of Mumford-Shah



*Chan & Vese '99, Implementation: D. Cremers*

# Multiphase Level Set Formulation

*Vese, Chan '02*



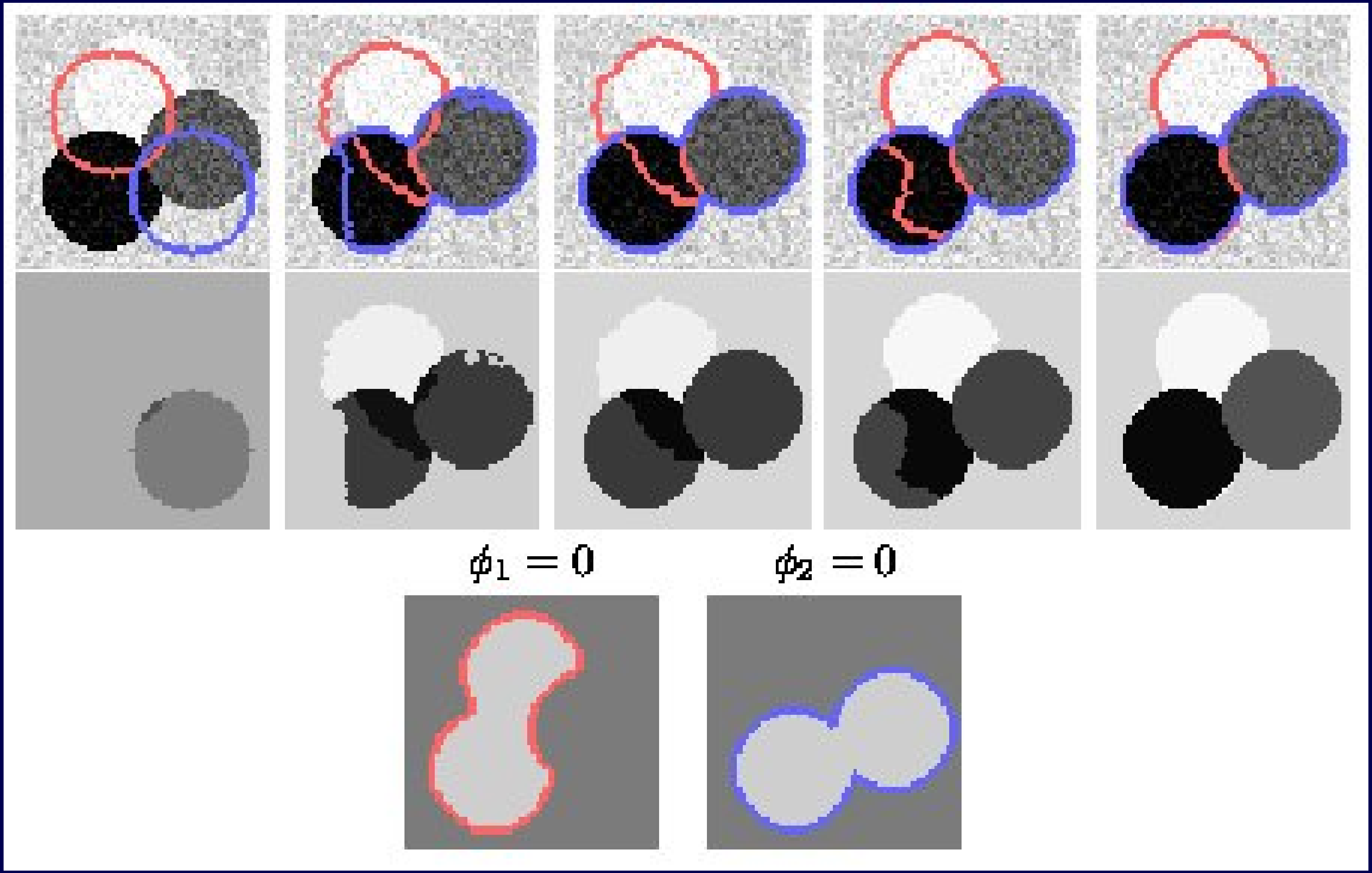

| | $\phi_1 \geq 0$ | $\phi_1 < 0$ |
|---|---|---|
| $\phi_2 \geq 0$ | $\Omega_1$ | $\Omega_2$ |
| $\phi_2 < 0$ | $\Omega_3$ | $\Omega_4$ |

$$E(\phi_1, \phi_2, u) = \int_\Omega (I - u_1)^2 H\phi_1 H\phi_2 + (I - u_2)^2 \left(1 - H\phi_1\right) H\phi_2 \, dx$$

$$+ \int_\Omega (I - u_3)^2 H\phi_1 \left(1 - H\phi_2\right) + (I - u_4)^2 \left(1 - H\phi_1\right)\left(1 - H\phi_2\right) dx$$

$$+ \nu \sum_i \int_\Omega |\nabla H\phi_i| dx \qquad \frac{\partial \vec{\phi}}{\partial t} = -\frac{dE}{d\vec{\phi}}$$

# Multiphase Level Set Formulation




*Chan, Vese ’01*

# Combined Mumford-Shah on Difference Image and Geodesic Active Contours

*Goldenberg, Kimmel, Rivlin, Rudzsky '05*

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Segmenting texture and motion information

Statistical shape priors for level set functions

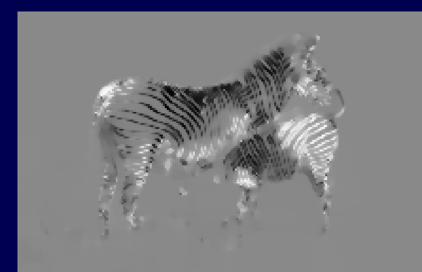
*Cremers, Rousson, Deriche, IJCV 2006*

*"A Review of Statistical Approaches to Level Set Segmentation: Integrating color, texture, motion and shape"*

# Texture Segmentation

# Texture Segmentation

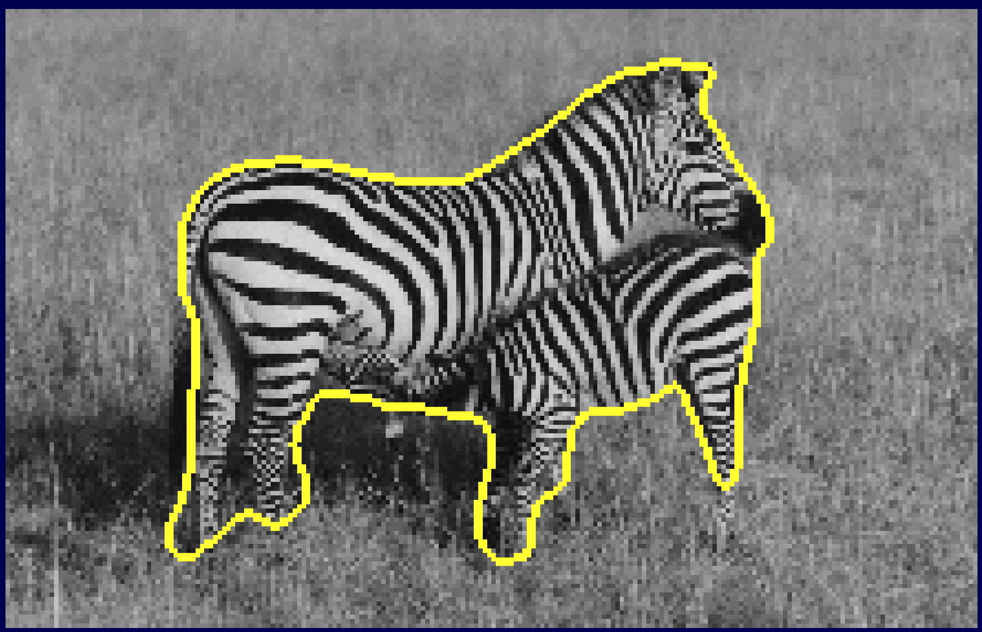
1. Generate sparse texture features by nonlinear diffusion filtering

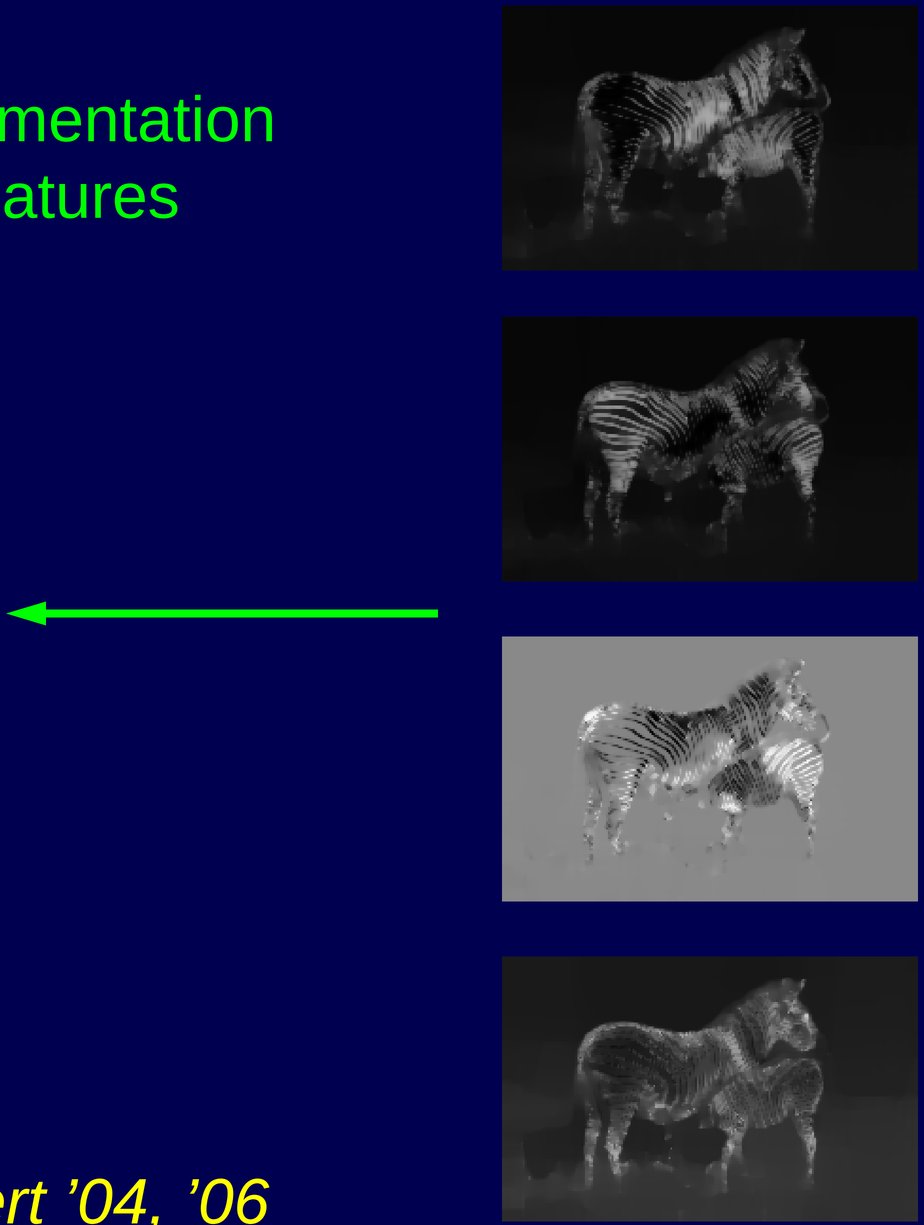


*Brox, Weickert ’04, ’06*

# Texture Segmentation

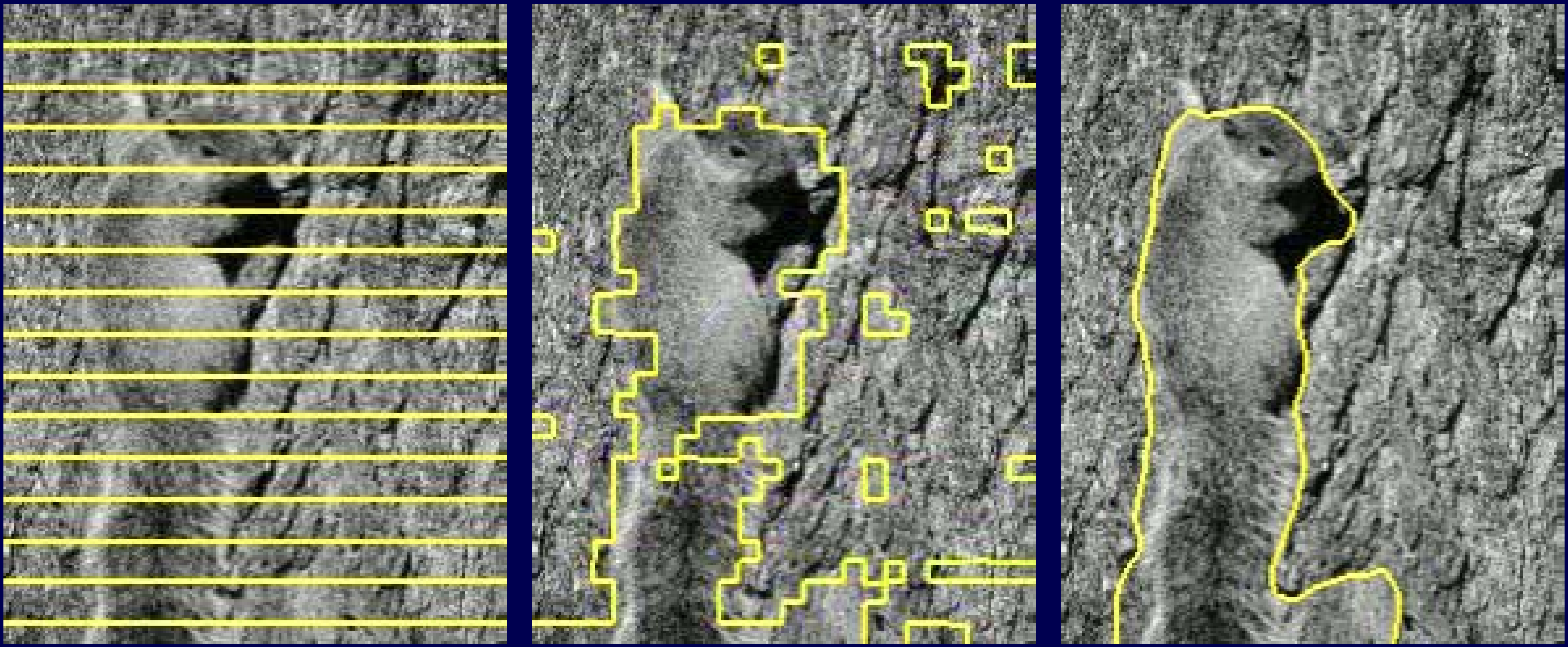
2. Mumford-Shah segmentation
of vector-valued features





*Brox, Weickert ’04, ’06*

# Texture Segmentation



efficient coarse-to-fine scheme

*Brox, Weickert '04, '06*

# Efficient Multiphase Formulation

2-phase solution

multiphase solution

*Brox, Weickert '04, '06*

# Efficient Multiphase Formulation

*Brox, Weickert '04, '06*

# Low-level Criteria for Segmentation

Intensity

Texture

Motion

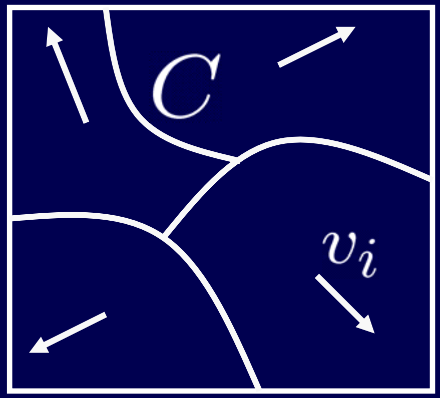
# Bayesian Motion Segmentation

Goal: Determine the most likely velocity field given the intensity gradients

Maximize $\mathcal{P}\,(v\,|\,\nabla_3 I) = \dfrac{\mathcal{P}\,(\nabla_3 I\,|\,v)\;\;\mathcal{P}(v)}{\mathcal{P}(\nabla_3 I)}$

where $\nabla_3 I \equiv (\nabla I, I_t)^\top$

$v(x) = v_i + \eta$ in $R_i$ $\qquad \mathcal{P}(v) \propto \exp\,(-\nu|C|)$




$$E(\{v_i\}, C) = -\sum_i \int_{R_i} \log \mathcal{P}(\nabla_3 I\,|\,v_i)\,dx \;+\; \nu\,|C|$$

$$E(\{v_i\}, C) = \sum_i \int_{R_i} \frac{|v_i^\top\,\nabla_3 I|^2}{|v_i|^2\,|\nabla I|^2}\,dx \;+\; \nu\,|C|$$

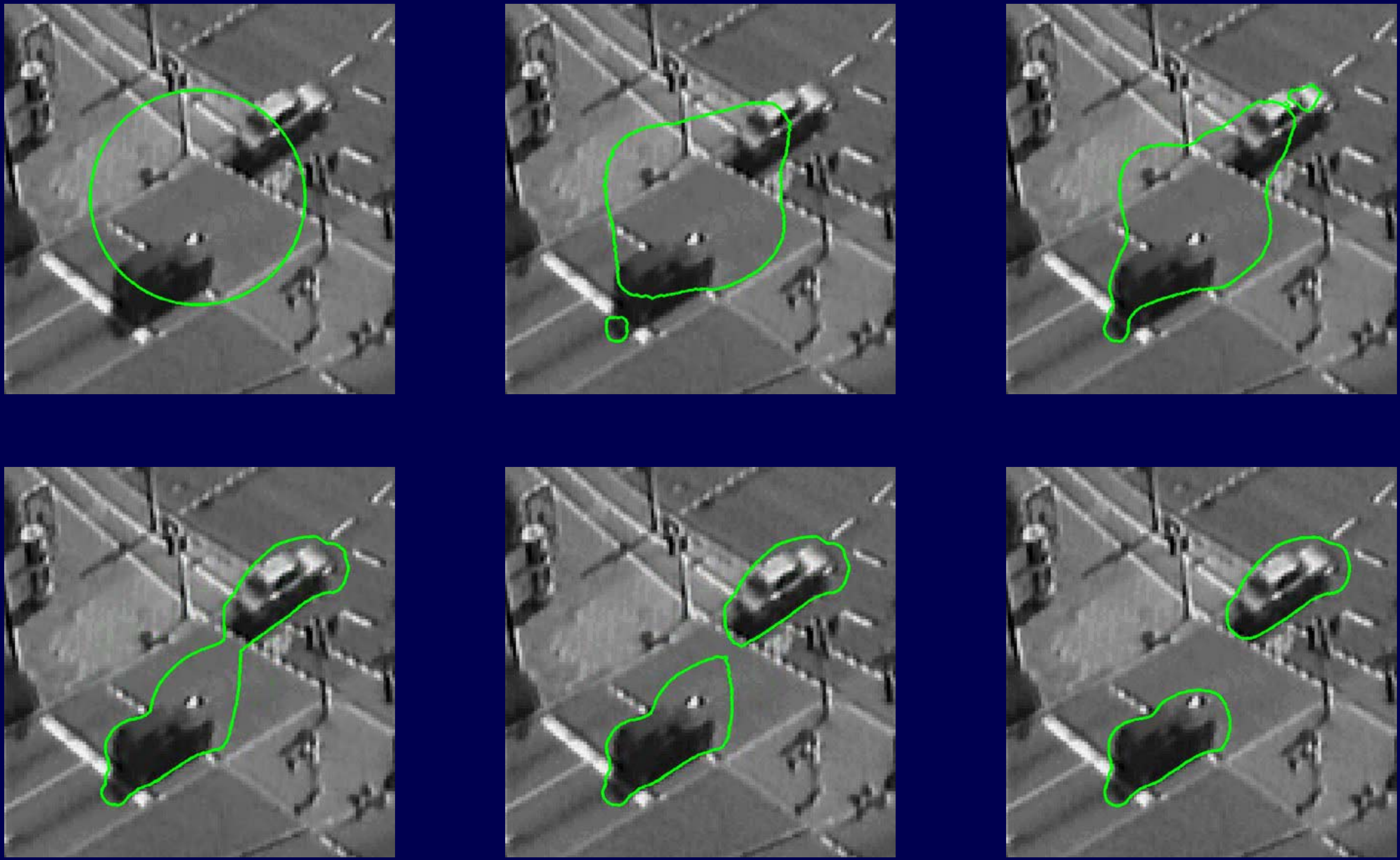
Segmentation & Motion Estimation

*Cremers, CVPR ’03, Cremers, Soatto IJCV ’05*

# Motion Competition

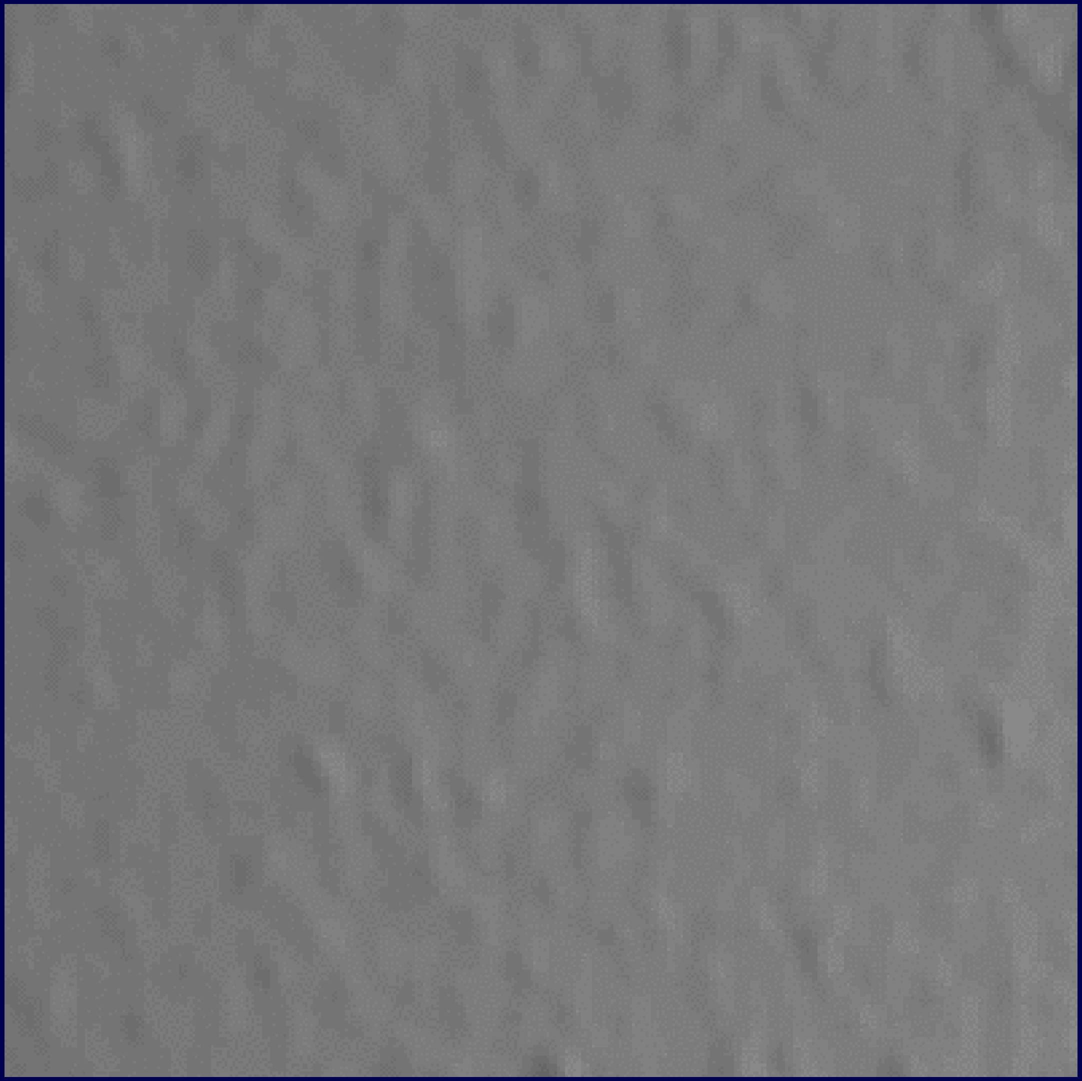
Original sequence data courtesy of D. Koller and H.-H. Nagel

# Motion Competition



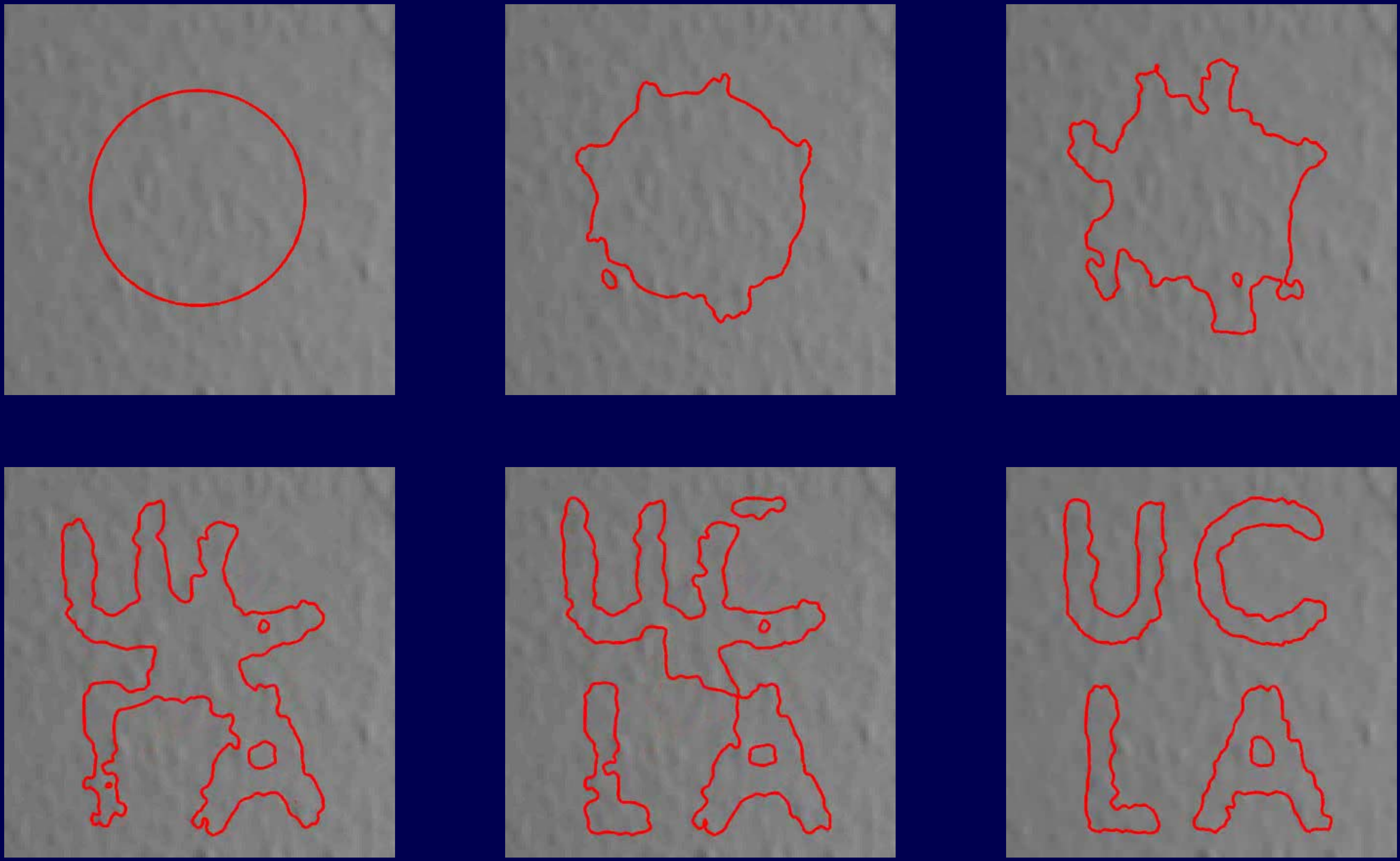
*Cremers, Yuille, '04*

# Motion Competition



What is moving?

# Motion Competition

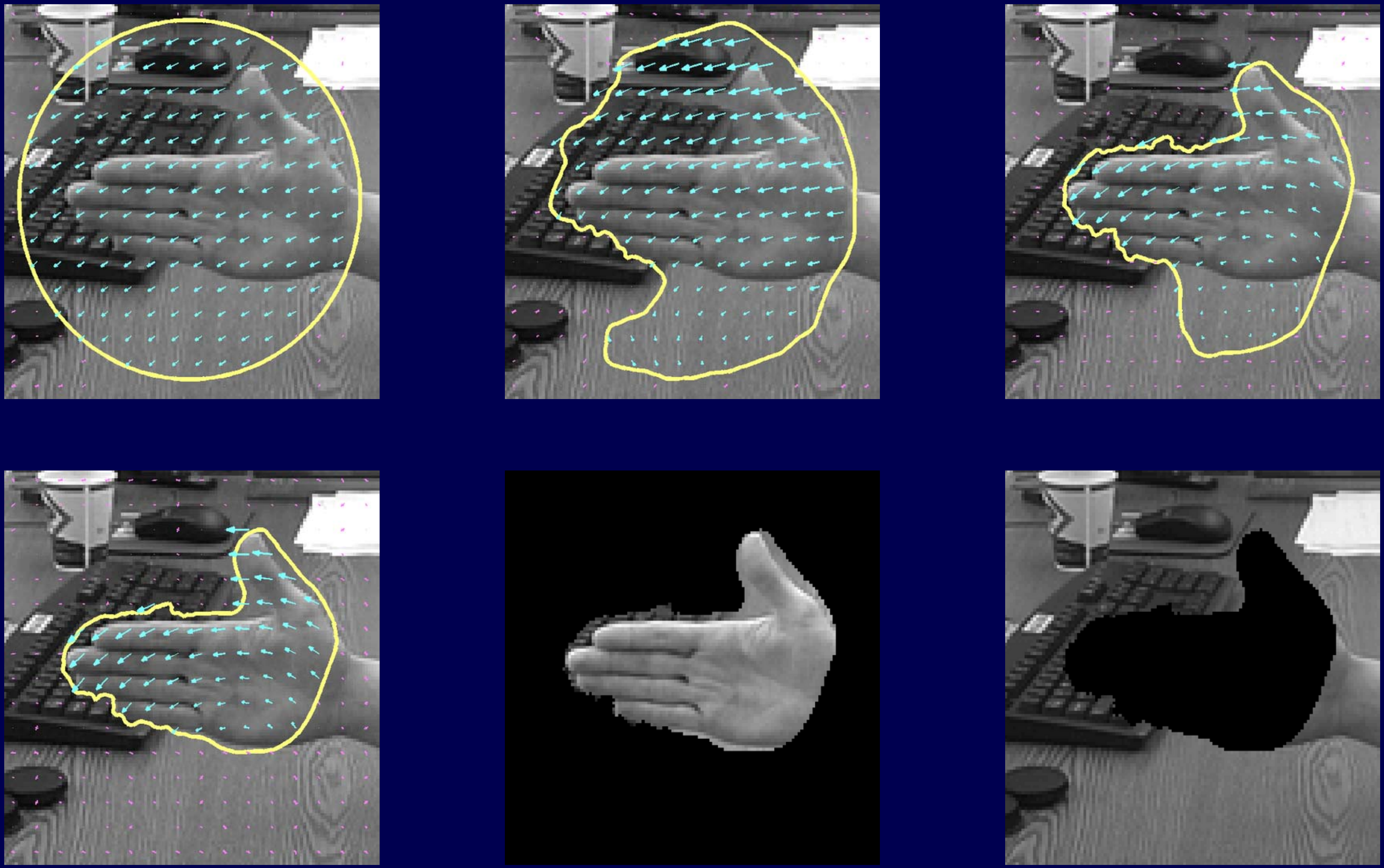


*Cremers, Yuille, '04*

# Motion Competition

Piecewise Parametric Motion

# Motion Competition

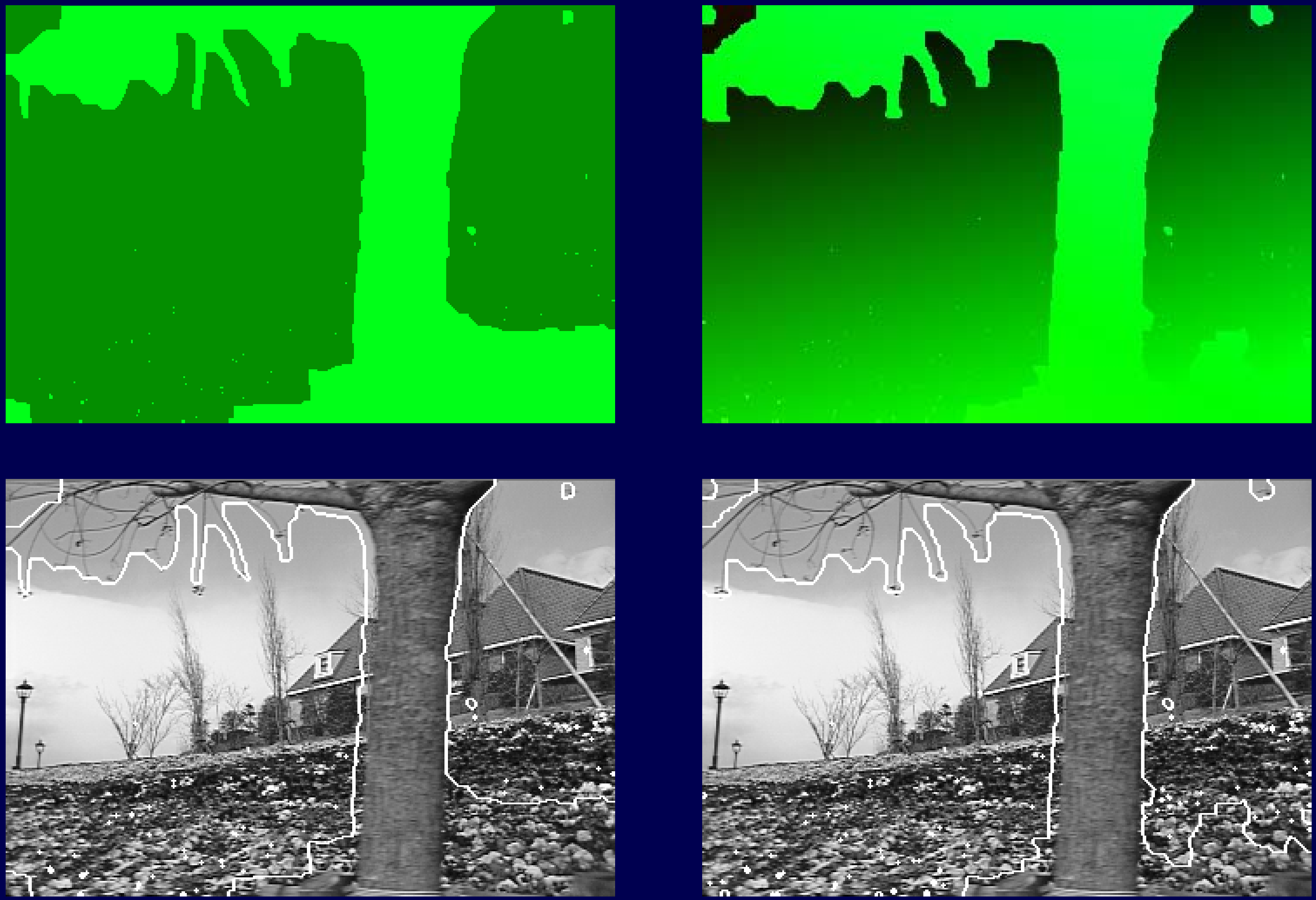


*Cremers, Soatto, IJCV '05:* Piecewise Parametric Motion

# Motion Competition via Graph Cuts

Image data courtesy of Wang & Adelson

# Motion Competition via Graph Cuts



piecewise constant piecewise affine

*Schoenmann & Cremers '06*

# Overview

Why level sets? Explicit vs. implicit contours

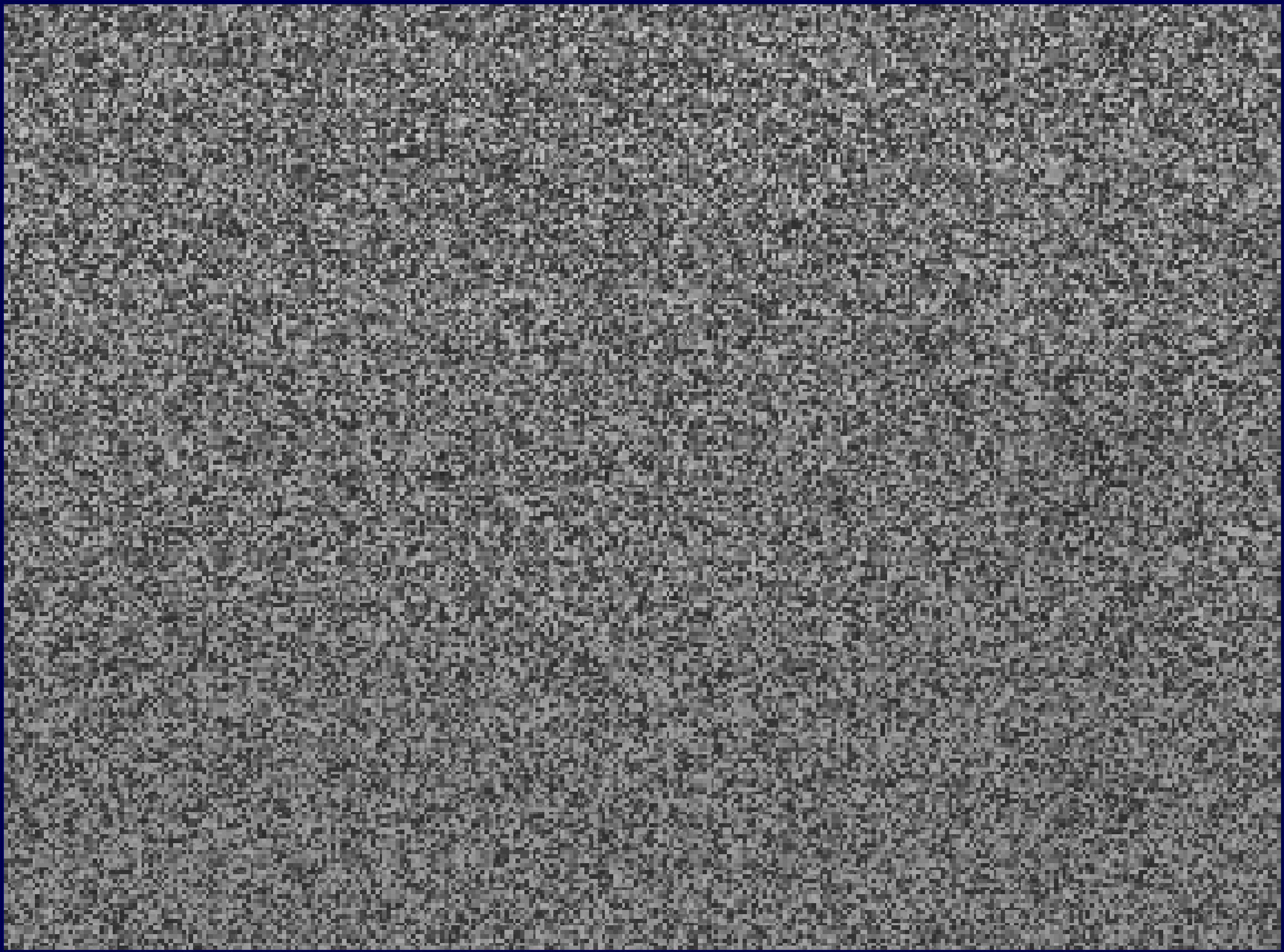
Some examples: graphics & 3D reconstruction

Level set methods for image segmentation

Segmenting texture and motion information

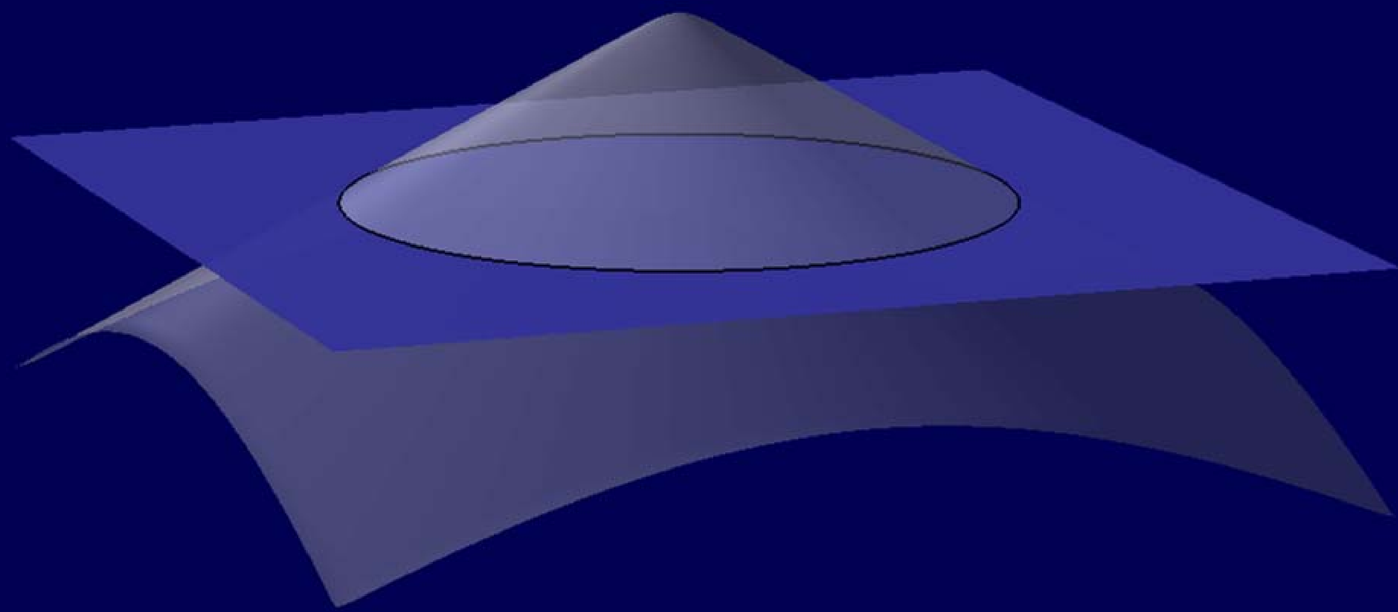
Statistical shape priors for level set functions

# Corrupted Low-level Information



Input sequence with 90% noise

# Shape Priors for Level Set Segmentation



Leventon, Grimson, Faugeras ’00, Tsai et al. ’01, Rousson, Paragios ’02
Charpiat, Faugeras, Keriven ’03, Cremers, Sochen, Schnörr ’03
Rousson, Paragios, Deriche ’03, Riklin-Raviv, Sochen, Kiryati ’04
Rathi, Vasvani et al. ’05, Cremers, Osher, Soatto ’06, Cremers ’06

# Level Set Segmentation as Bayesian Inference

Find the most likely level set function given the image by maximizing the conditional probability

$$\mathcal{P}(\phi \,|\, I) = \frac{\mathcal{P}(I \,|\, \phi)\, \mathcal{P}(\phi)}{\mathcal{P}(I)}$$

This is equivalent to minimizing its negative logarithm:

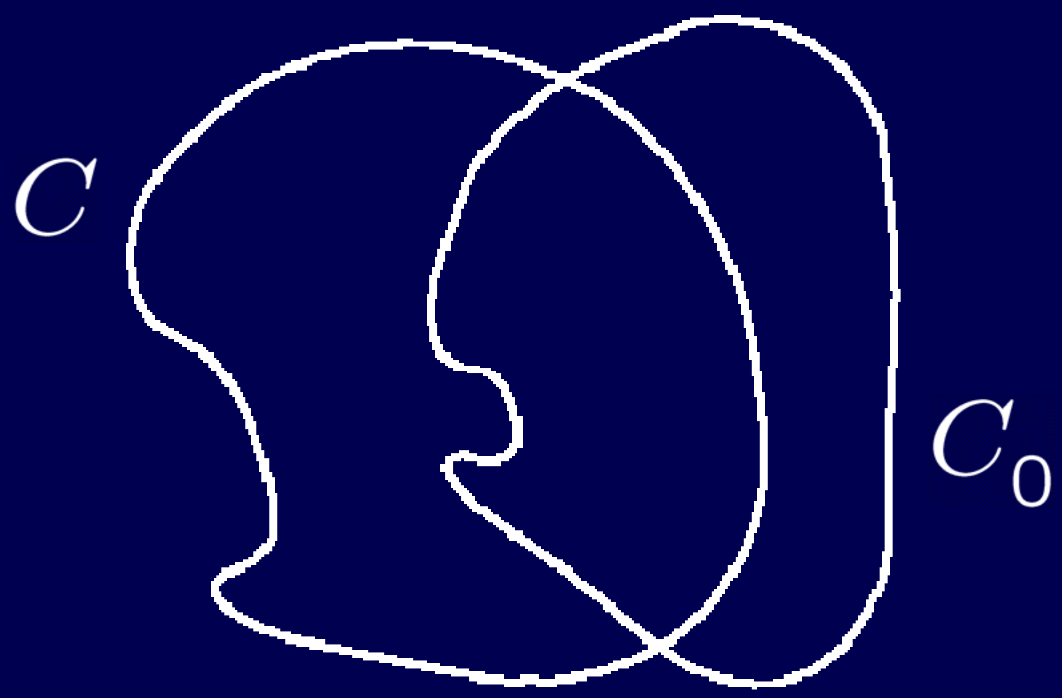
$$\begin{aligned} E(\phi, I) &= -\log\, \mathcal{P}(\phi | I) \\ &= -\log \mathcal{P}(I \,|\, \phi) - \log \mathcal{P}(\phi) \\ &= E_{image}(\phi, I) + E_{shape}(\phi) \end{aligned}$$

*Cremers, Osher, Soatto, Int. J. of Computer Vision '06*

# Shape Distances for Level Sets

$$C = \{x \mid \phi(x) = 0\}, \quad C_0 = \{x \mid \phi_0(x) = 0\}$$

$$d^2(\phi, \phi_0) = \int (H\phi(x) - H\phi_0(x))^2 \, dx$$




$$H\phi = \begin{cases} 1, \text{if } \phi \geq 0 \\ 0, \text{else} \end{cases}$$

# Shape Distances for Level Sets

$$C = \{x \mid \phi(x) = 0\}, \quad C_0 = \{x \mid \phi_0(x) = 0\}$$

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$$H\phi = \begin{cases} 1, \text{if } \phi \geq 0 \\ 0, \text{else} \end{cases}$$


*Zhu,Chan ’03, Charpiat et al. ‘04, Riklin-Raviv et al. ’04*

# Invariance by Intrinsic Alignment

$$C = \{x \mid \phi(x) = 0\}, \quad C_0 = \{x \mid \phi_0(x) = 0\}$$

$$d^2(\phi, \phi_0) = \int \left(H\phi\left(\sigma_\phi x + \mu_\phi\right) - H\phi_0(x)\right)^2 dx$$

where: $\mu_\phi = \int x\, h\phi\, dx, \quad h\phi = \frac{H\phi}{\int H\phi\, dx}$

center of gravity

and $\sigma_\phi = \left(\int (x - \mu_\phi)^2\, h\phi\, dx\right)^{1/2}$

intrinsic scale

→ Closed-form solution, accurate shape gradients

*Cremers, Osher, Soatto, Int. J. of Computer Vision '06*

# Invariance to Translation and Scaling

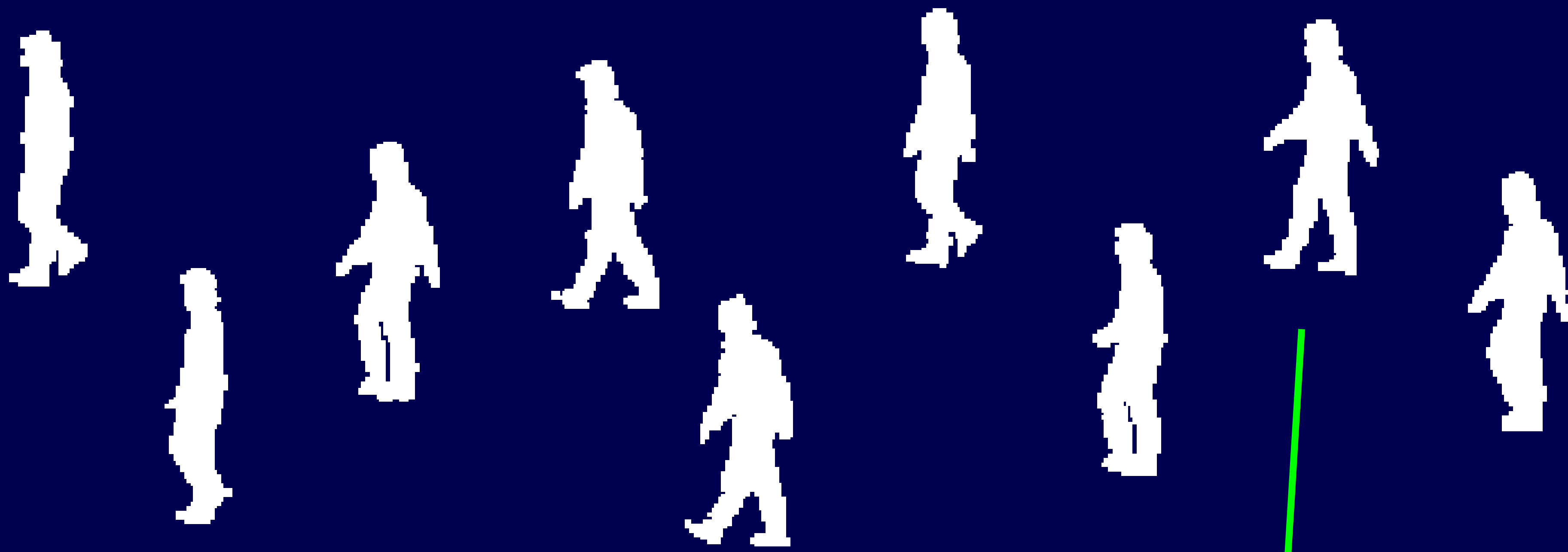
zero level of normalized surface

small medium large

$$E(\phi) = E_{image}(\phi) + \alpha\, d^2(\phi, \phi_0)$$

*Cremers, Osher, Soatto, Int. J. of Computer Vision '06*

# Training Shapes

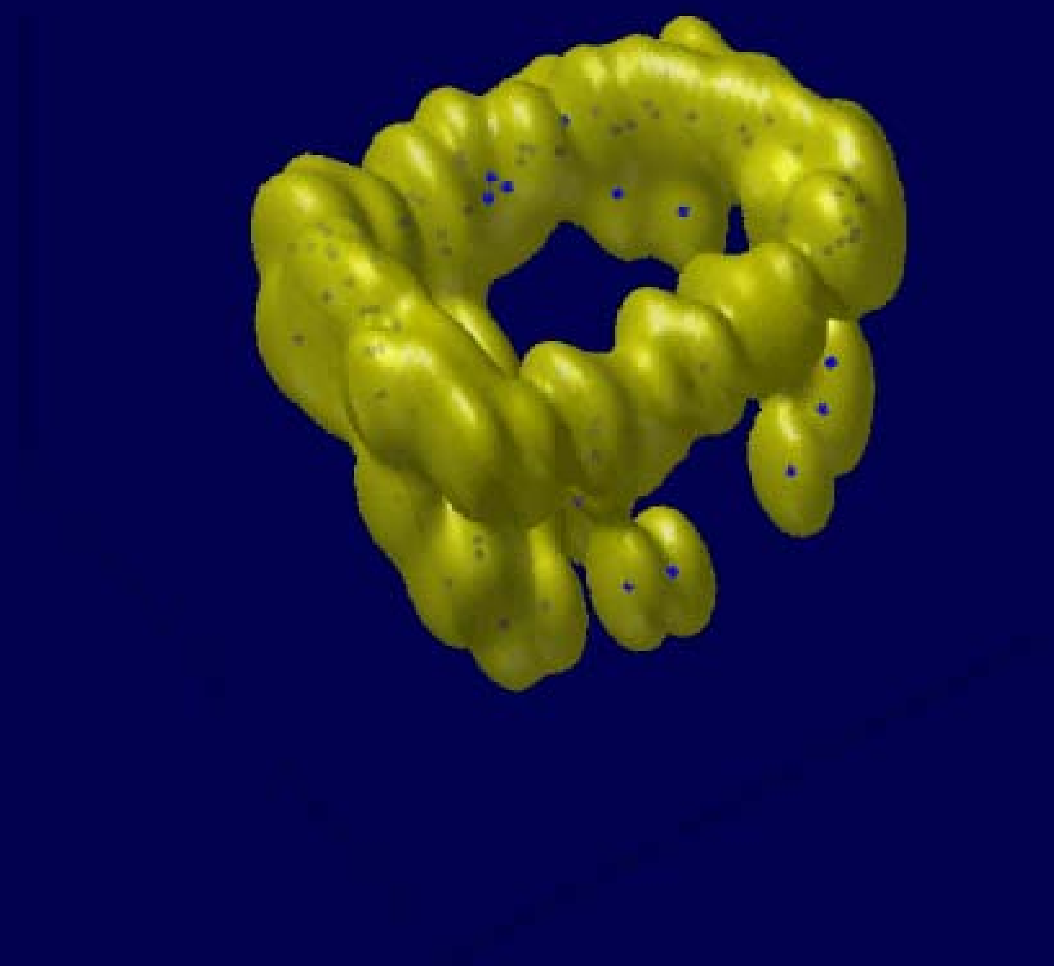


$$d^2(\phi, \phi_0) = \int \Big(H\phi\big(\sigma_\phi\, x + \mu_\phi\big) - H\phi_0(x)\Big)^2\, dx$$

$$\mathcal{P}(\phi) \propto \frac{1}{N}\sum_{i=1}^{N} \exp\left(-\frac{1}{2\,\sigma^2}\, d^2(\phi, \phi_i)\right)$$

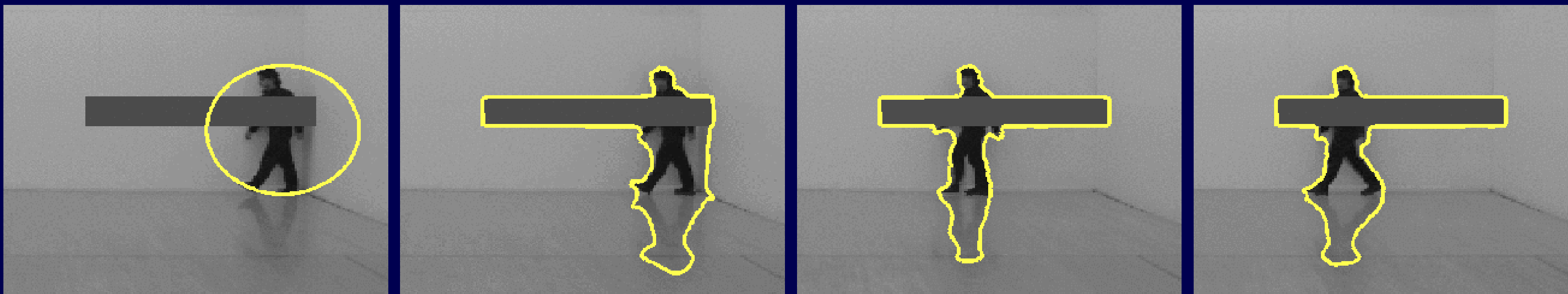
*Cremers, Osher, Soatto, Int. J. of Computer Vision '06*

# Kernel Density Estimation for Level Sets

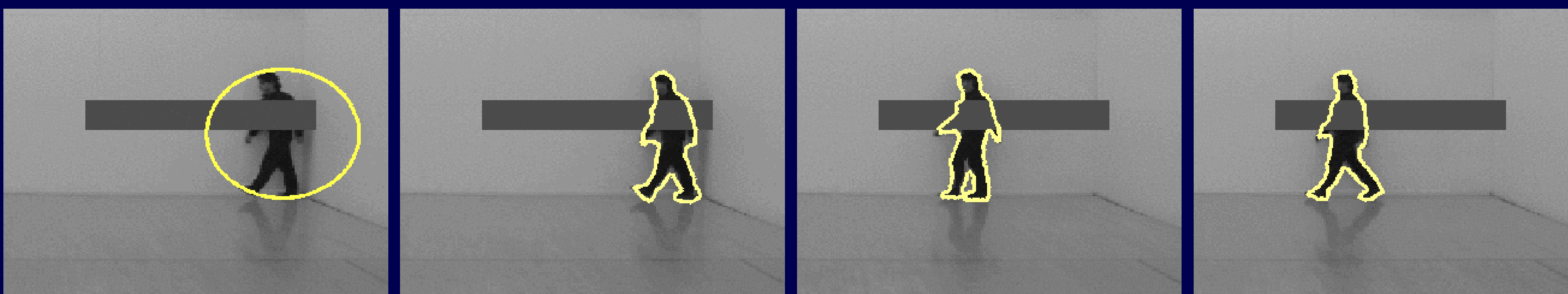


Estimated Density in 3D

# Kernel Density Estimation for Level Sets



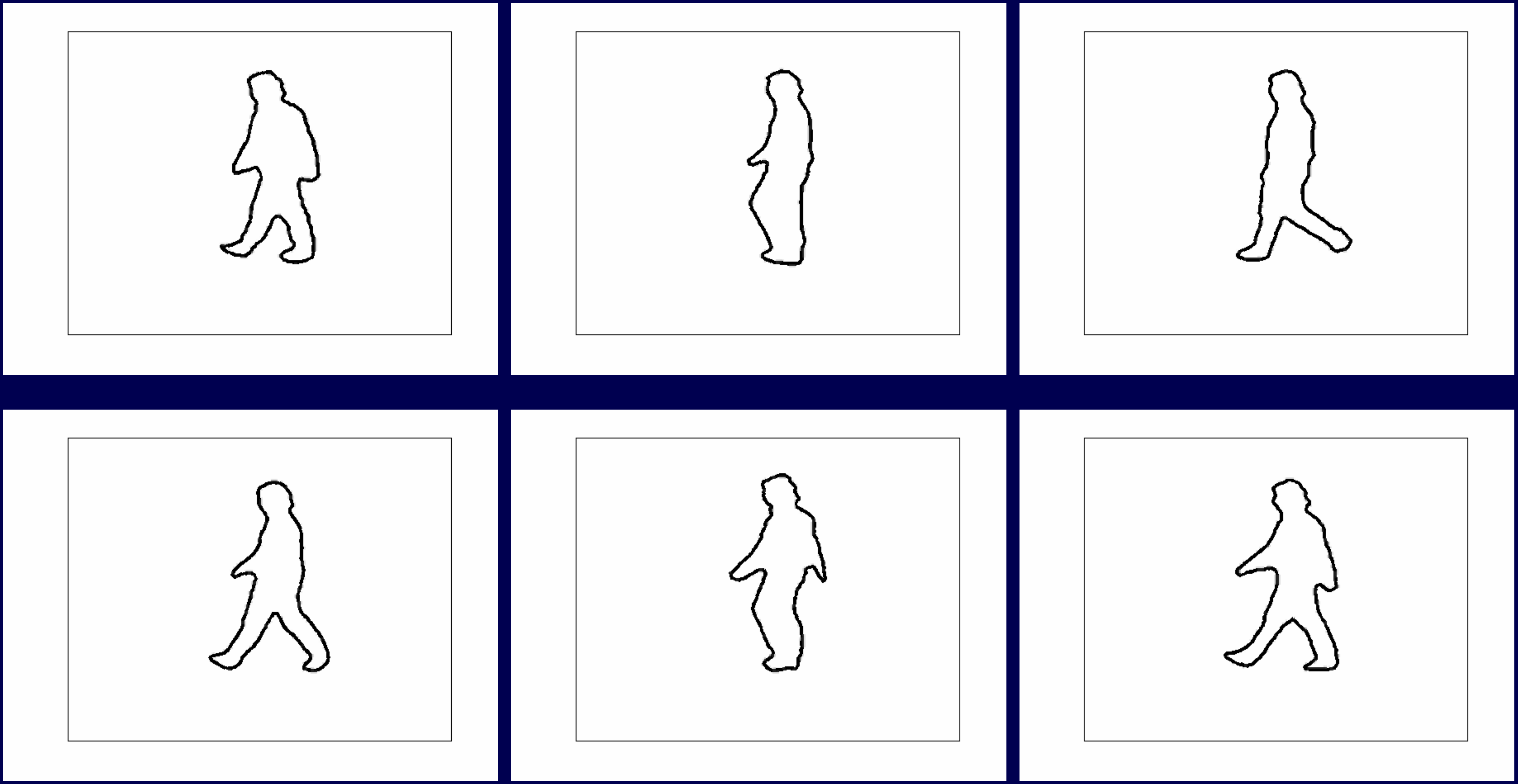
Purely geometric prior



Nonparametric prior

*Cremers, Osher, Soatto, IJCV '06*

# Dynamical Models for Implicit Shapes



Training sequence

# Dynamical Models for Implicit Shapes

1. Low-dim. representation via PCA (*Leventon et al. '00, Tsai et al. '01*):

$$\phi_i(x) \approx \phi_0(x) + \sum_{j=1}^{m} \alpha_{ij} \psi_j(x)$$

mean — eigenmodes

$$\alpha_{ij} = \int (\phi_i - \phi_0)\, \psi_j \, dx \qquad \boldsymbol{\alpha}_i = (\alpha_{i1}, \ldots, \alpha_{im})$$

2. Autoregressive model for the shape coefficients:

$$\boldsymbol{\alpha}_t = \boldsymbol{\mu} + \sum_{i=1}^{k} A_i \, \boldsymbol{\alpha}_{t-i} + \boldsymbol{\eta}$$

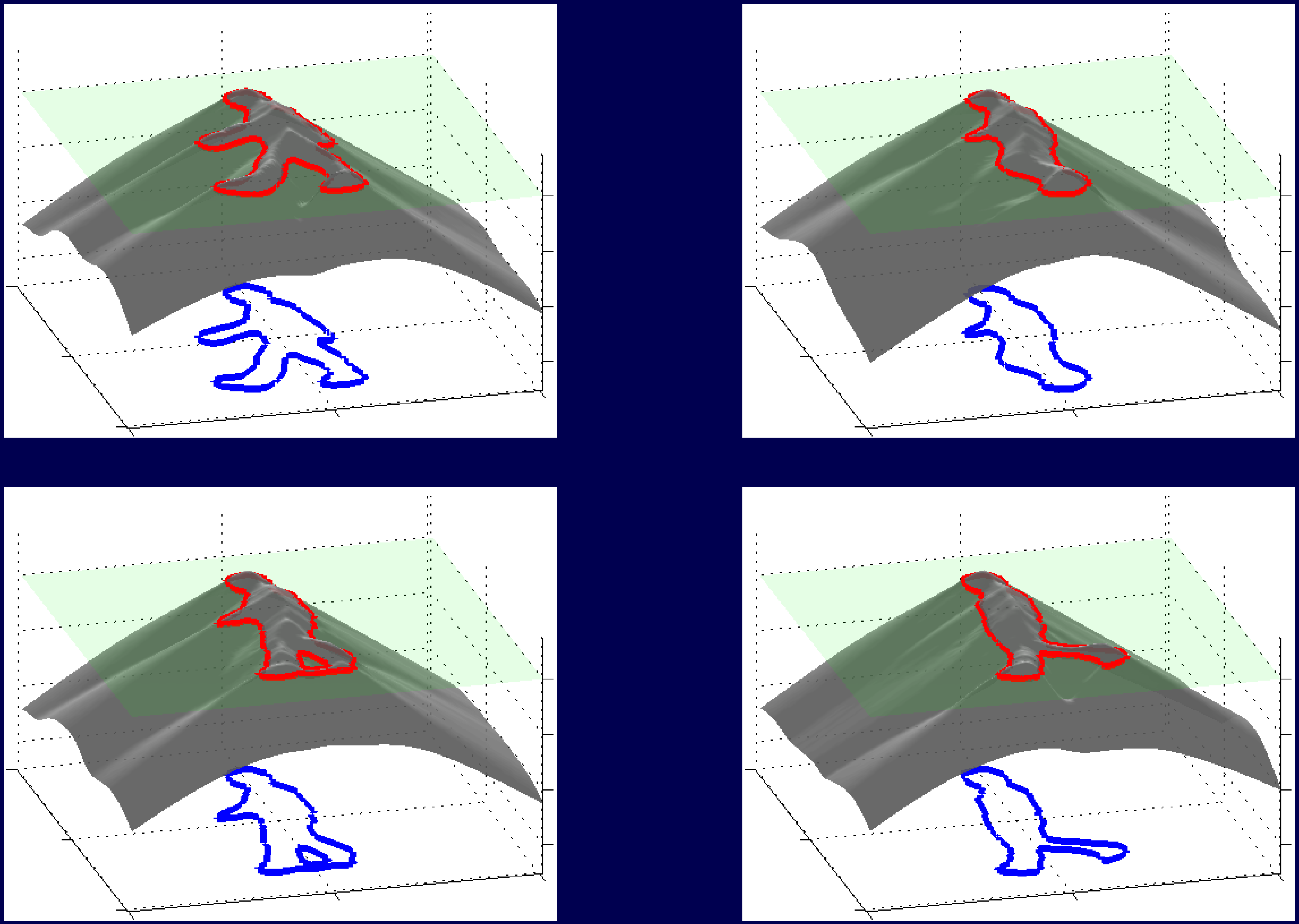
mean — transition matrices — Gaussian noise

3. Synthesize shape vectors and embedding surfaces:

$$\phi_t(x) = \phi_0(x) + \int \boldsymbol{\alpha}_t^\top \, \boldsymbol{\psi}(x)\, dx$$

*Cremers, IEEE Trans. on PAMI '06*

# Statistically Sampled Embedding Functions



*Cremers, IEEE Trans. on PAMI '06*

# Dynamical Priors for Level Set Tracking

Bayesian Aposteriori Maximization:

$$\hat{\boldsymbol{\alpha}}_t = \arg\max_{\boldsymbol{\alpha}_t} \mathcal{P}\left(\boldsymbol{\alpha}_t \,|\, I_t, \hat{\boldsymbol{\alpha}}_{1:t-1}\right)$$

input image

previous shape estimates

$$\mathcal{P}\left(\boldsymbol{\alpha}_t \,|\, I_t, \hat{\boldsymbol{\alpha}}_{1:t-1}\right) \propto \mathcal{P}\left(I_t \,|\, \boldsymbol{\alpha}_t, \hat{\boldsymbol{\alpha}}_{1:t-1}\right) \; \mathcal{P}\left(\boldsymbol{\alpha}_t \,|\, \hat{\boldsymbol{\alpha}}_{1:t-1}\right)$$

*Cremers, IEEE Trans. on PAMI '06*

# Dynamical Priors for Level Set Tracking

Bayesian Aposteriori Maximization:

$$\hat{\alpha}_t = \arg\max_{\alpha_t} \mathcal{P}\left(\alpha_t \,|\, I_t, \hat{\alpha}_{1:t-1}\right)$$

$$\mathcal{P}\left(\alpha_t \,|\, I_t, \hat{\alpha}_{1:t-1}\right) \propto \boxed{\mathcal{P}\left(I_t \,|\, \alpha_t\right)} \boxed{\mathcal{P}\left(\alpha_t \,|\, \hat{\alpha}_{1:t-1}\right)}$$

Image formation model (color, texture, motion,…)

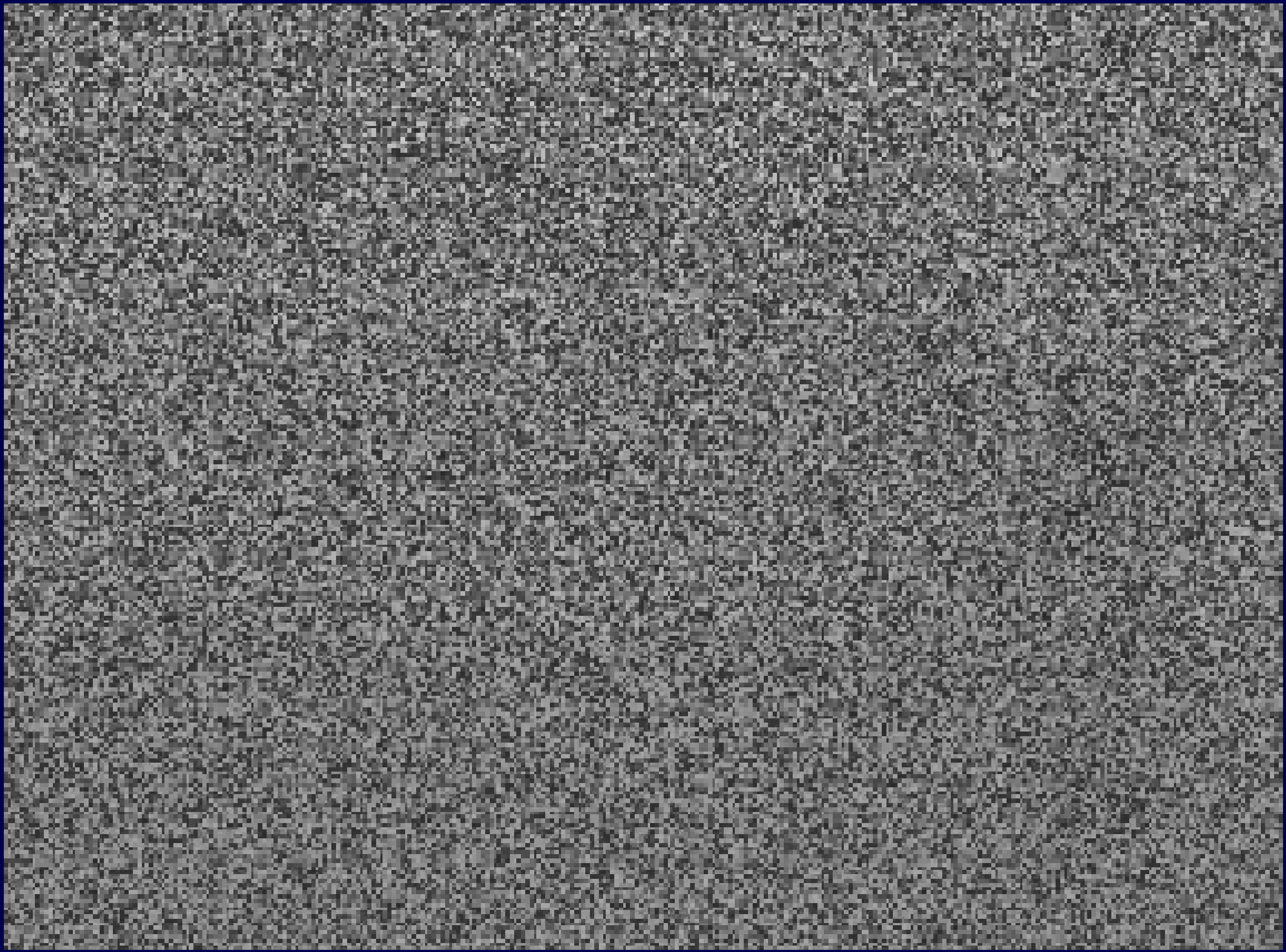
Dynamical shape model

$$E = -\log \mathcal{P}(\alpha_t | I_t, \hat{\alpha}_{1:t-1}) = E_{dat}(\alpha_t, I_t) + \boxed{E_{dyn}(\alpha_t, \hat{\alpha}_{1:t-1})}$$

Optimization by gradient descent: $\frac{d\alpha_t}{d\tau} = -\frac{\partial E}{\partial \alpha_t}$

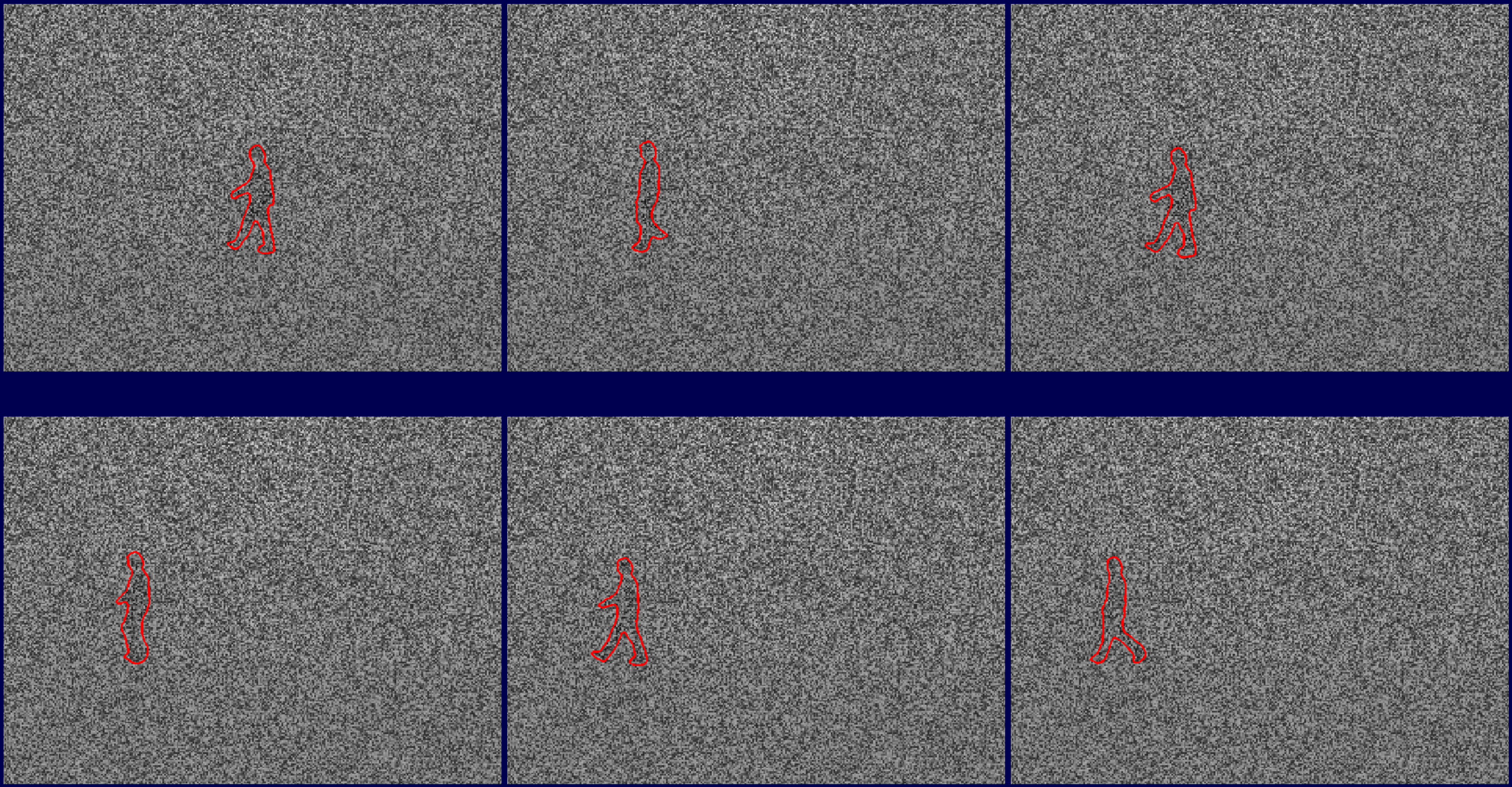
*Cremers, IEEE Trans. on PAMI ’06*

# Dynamical Priors for Level Set Tracking



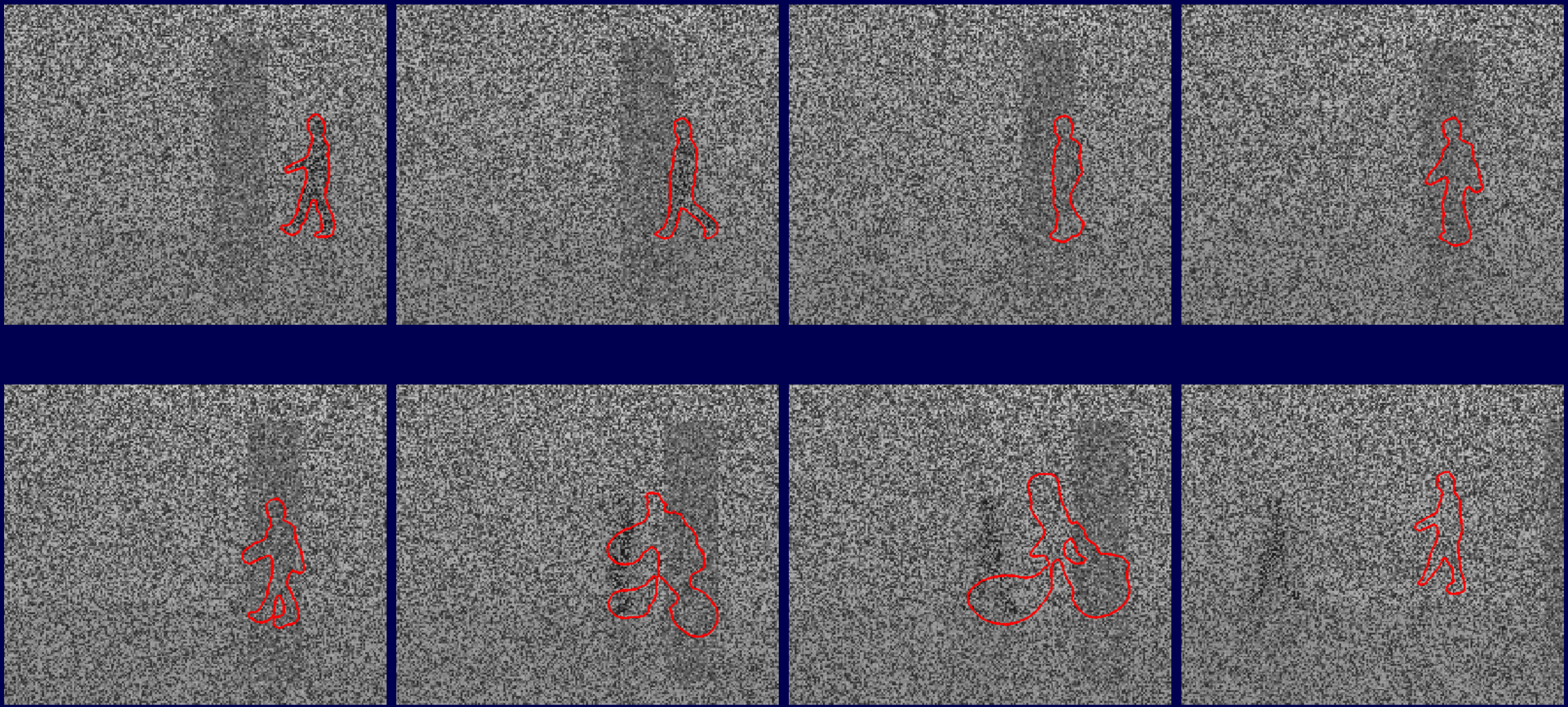
Input sequence with 90% noise

# Dynamical Priors for Level Set Tracking



*Cremers, IEEE Trans. on PAMI '06*

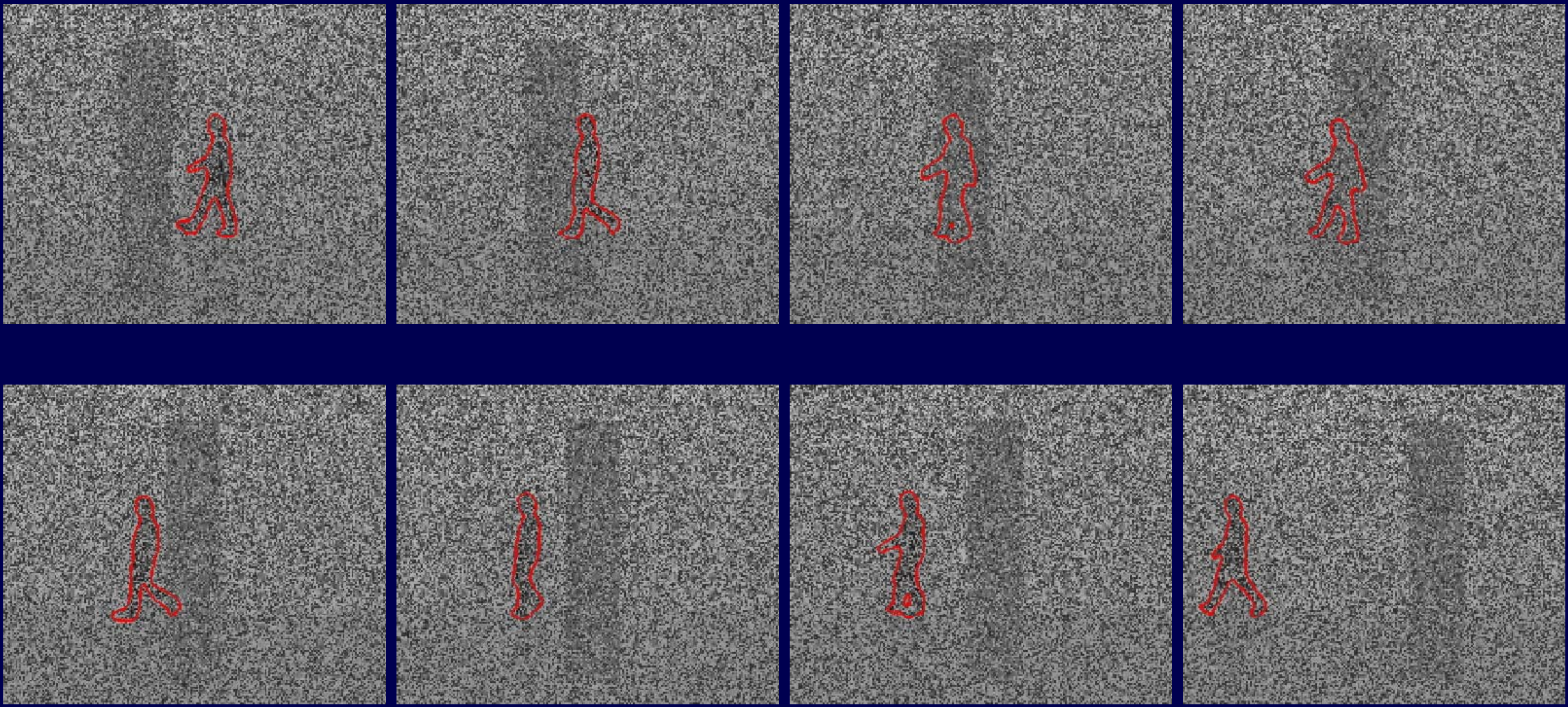
# Tracking through an Occlusion



Pure deformation model

*Cremers, IEEE Trans. on PAMI '06*

# Tracking through an Occlusion



Model of joint deformation and transformation

*Cremers, IEEE Trans. on PAMI '06*

# Multiple Known Objects

# Multiple Known Objects

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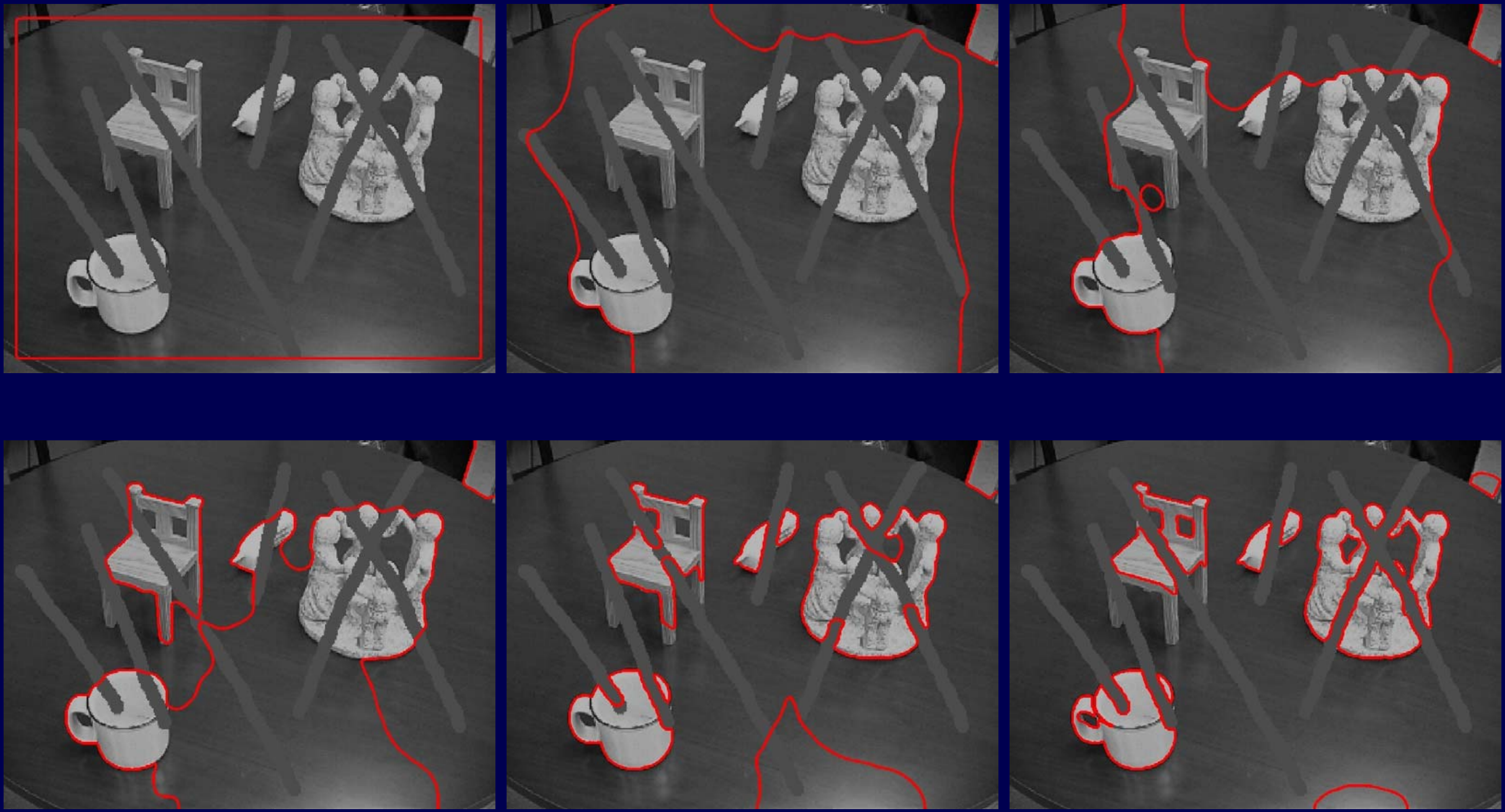
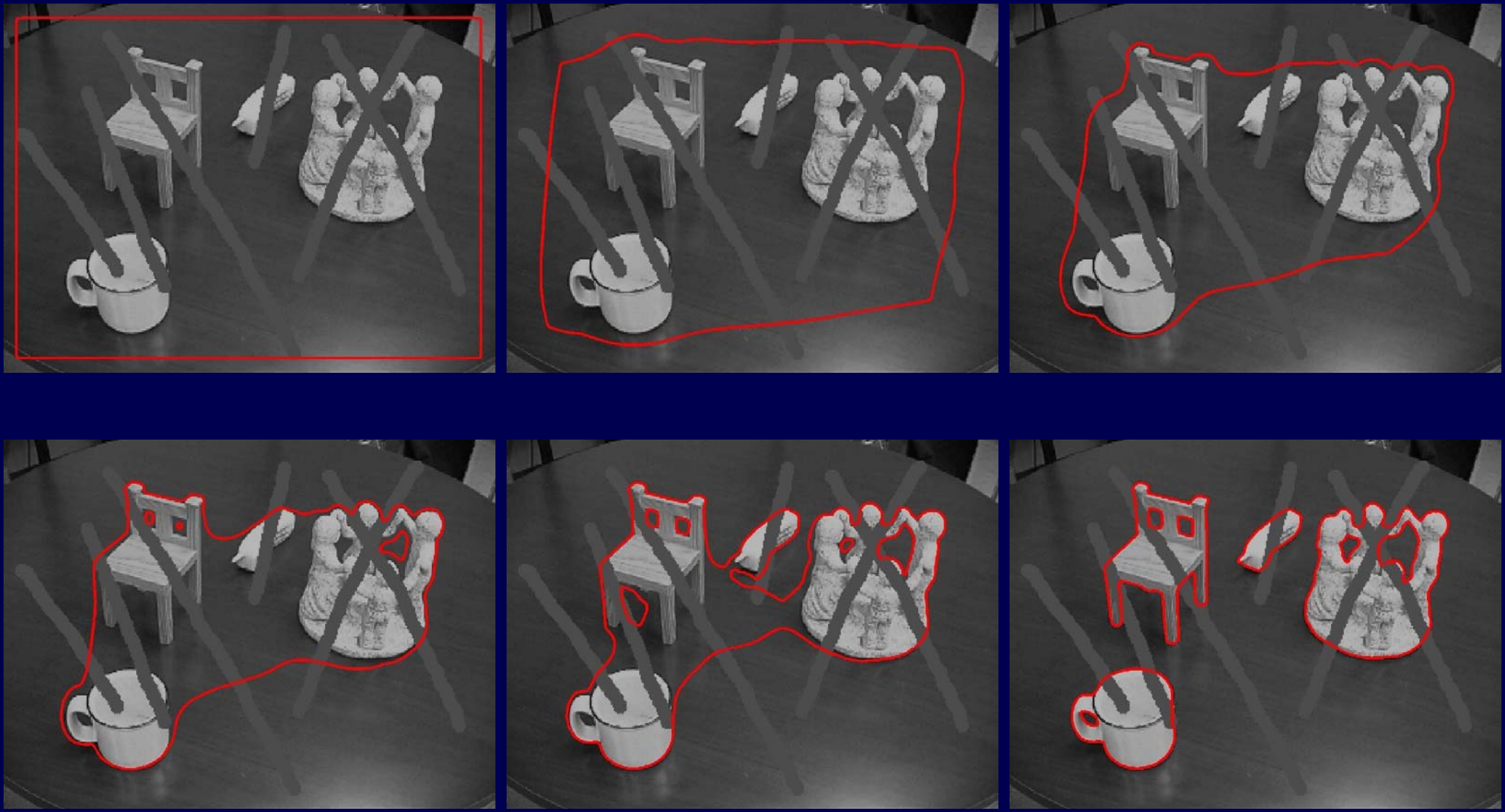


*Chan, Vese, TIP '01,* Author: D. Cremers

# Recognition-driven Segmentation



*Cremers, Sochen, Schnörr, ECCV '04, IJCV '06*

# Multiphase Dynamic Labeling

$$\mathbf{L} : \Omega \to \mathbf{R}^n \qquad \mathbf{L}(x) = \Big( L_1(x), \ldots, L_n(x) \Big)$$

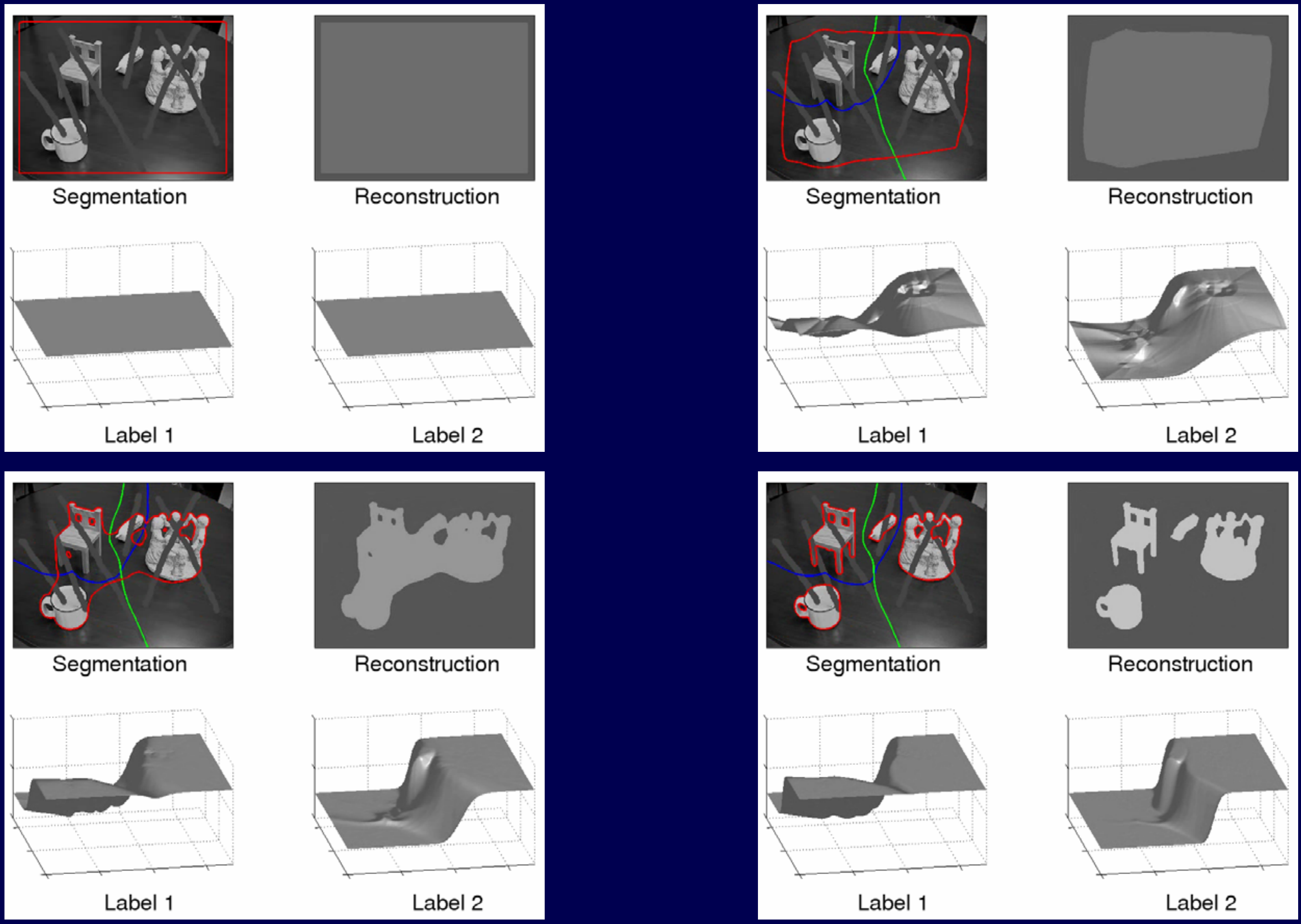
$$E_{shape}(\phi, \mathbf{L}) = \sum_{i=1}^{2^n} \int \big(\phi - \phi_i\big)^2 \chi_i(\mathbf{L})\, dx + \gamma \sum_{j=1}^{n} \int |\nabla L_j| dx$$

$$\chi_i(\mathbf{L}) = \frac{1}{4^n} \prod_{j=1}^{n} (L_j + \ell_j)^2, \qquad \ell_j \in \{-1, +1\}$$

→ Joint segmentation and recognition

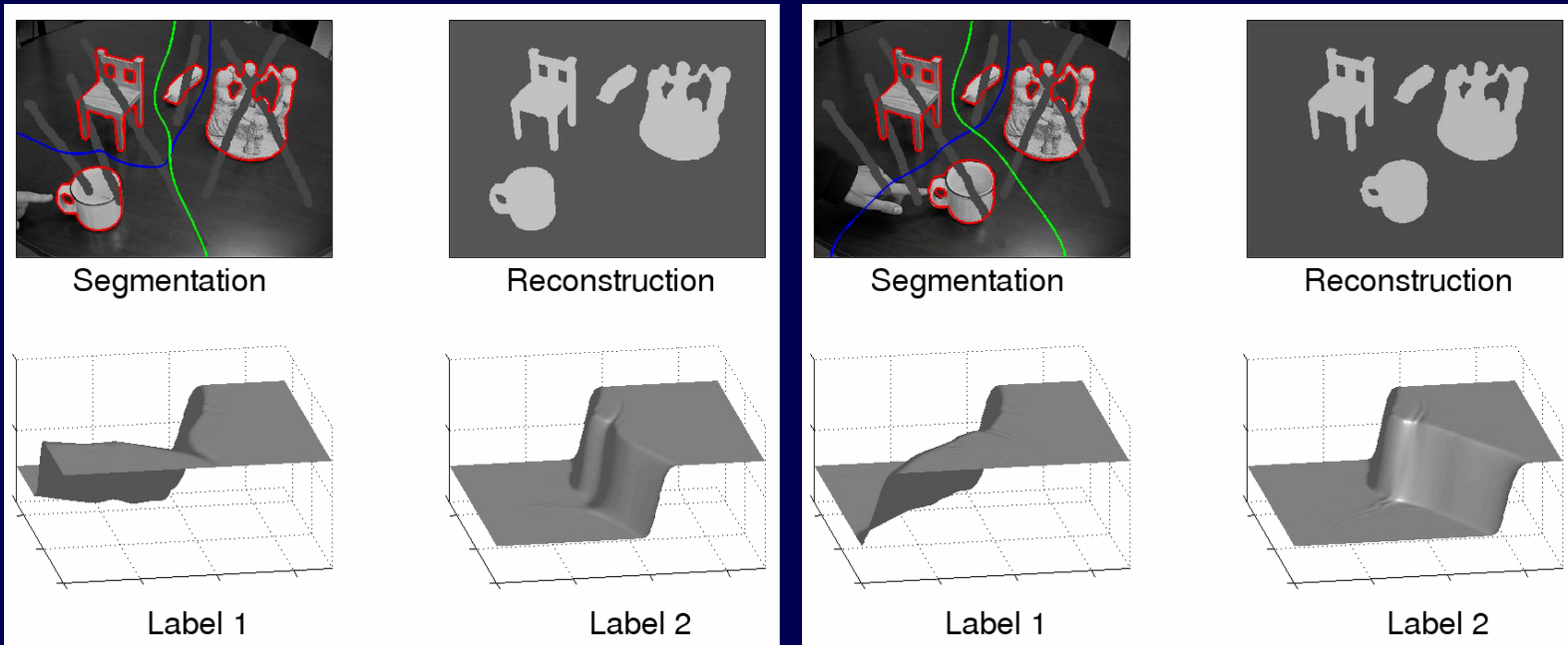
*Cremers, Sochen, Schnörr, ECCV ’04, IJCV ’06*

# Multiphase Dynamic Labeling




*Cremers, Sochen, Schnörr, ECCV '04, IJCV '06*

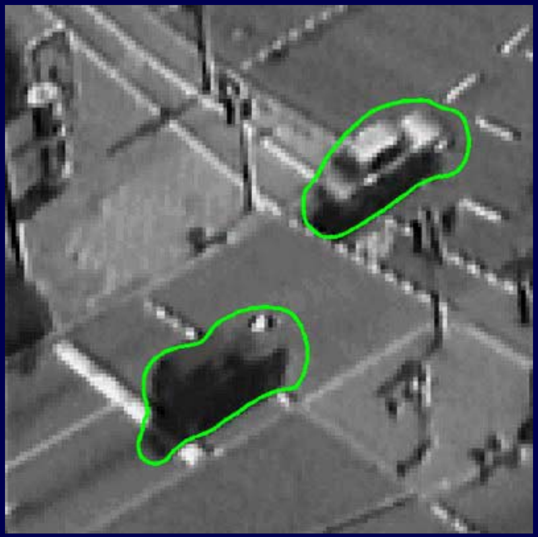
# Multiphase Dynamic Labeling




Changes in the input data induce an update of the labeling functions and the embedded decision boundaries.
Energy minimization leads to joint segmentation and recognition.

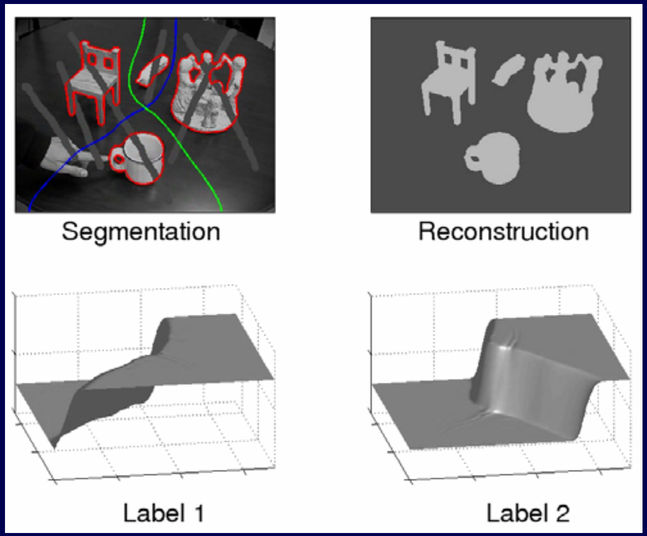
*Cremers, Sochen, Schnörr, ECCV '04, IJCV '06*

# Level Set Methods in Computer Vision

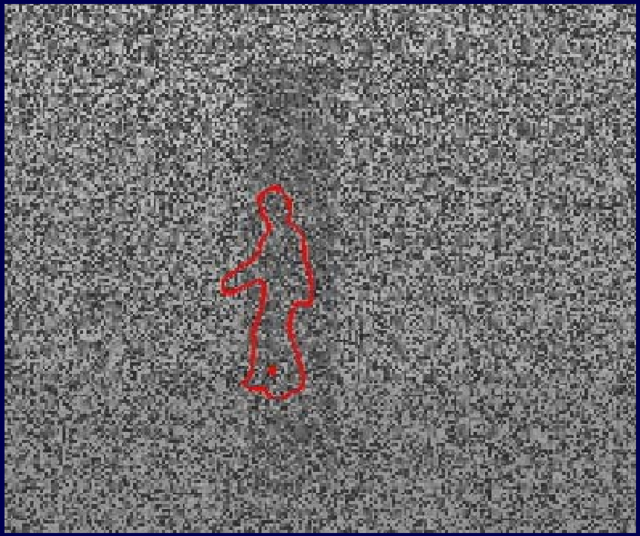


Motion Competition

Multiview 3D Segmentation




Recognition Modeling



Dynamical Shape Priors